Fast and Efficient Algorithms in Computational Electromagnetics
Fast and Efficient Algorithms in Computational Electromagnetics

Weng Cho Chew, Jian-Ming Jin, Eric Michielssen, Jiming Song, eds.

Artech House, INC.

Boston / London
To our wives and our parents
# Contents

Preface xix  
Acknowledgments xxv  

1 Introduction to Electromagnetic Analysis and Computational Electromagnetics 1  
1.1 Introduction 1  
1.2 A Bit of History 4  
1.3 More on Differential Equation Solvers 9  
  1.3.1 Convergence Rate of Iterative Differential Equation Solvers 10  
1.4 Integral Equation Solvers 13  
  1.4.1 Surface Integral Equations 13  
  1.4.2 The Internal Resonance Problem 15  
  1.4.3 Volume Integral Equation 19  
  1.4.4 Green’s Function 19  
  1.4.5 Method of Moments 20  
  1.4.6 Fast Integral Equation Solvers 21  
1.5 A Simplified View of the Multilevel Fast Multipole Algorithm 22  
1.6 Conclusion 26  
References 26  

2 Fast Multipole Method and Multilevel Fast Multipole Algorithm in 2D 39  
2.1 Introduction 39  
2.2 Introduction to Fast Multipole in 2D 39  
  2.2.1 A 2D MOM Problem 40  
  2.2.2 Addition Theorem for Bessel Functions 42  
  2.2.3 An Inefficient Factorization of the Green’s Function 44  
  2.2.4 Diagonalization of the Translation Operator 46  
  2.2.5 Summary and Hindsight 49  
  2.2.6 An Alternative Derivation of the Diagonalized Translator 49  
  2.2.7 Physical Interpretation of Aggregation, Translation, and Disaggregation 50  
  2.2.8 Bandwidth of the Radiation Pattern 51  
  2.2.9 Error Control 52  

<table>
<thead>
<tr>
<th>Chapter</th>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.3</td>
<td>Motivation for Multilevel Method</td>
<td>53</td>
</tr>
<tr>
<td></td>
<td>2.3.1 Factorization of the Green’s Function</td>
<td>54</td>
</tr>
<tr>
<td>2.4</td>
<td>The Multilevel Fast Multipole Algorithm</td>
<td>55</td>
</tr>
<tr>
<td></td>
<td>2.4.1 The Aggregation Process</td>
<td>56</td>
</tr>
<tr>
<td></td>
<td>2.4.2 Translation and Disaggregation</td>
<td>58</td>
</tr>
<tr>
<td></td>
<td>2.4.3 More on Interpolation and Anterpolation</td>
<td>59</td>
</tr>
<tr>
<td></td>
<td>2.4.4 Computational Complexity of MLFMA</td>
<td>61</td>
</tr>
<tr>
<td>2.5</td>
<td>Interpolation Error</td>
<td>63</td>
</tr>
<tr>
<td></td>
<td>2.5.1 Global Interpolation (Exact)</td>
<td>63</td>
</tr>
<tr>
<td></td>
<td>2.5.2 Local Interpolation (Exponentially Accurate)</td>
<td>65</td>
</tr>
<tr>
<td>2.6</td>
<td>FMM and Group Theory</td>
<td>67</td>
</tr>
<tr>
<td></td>
<td>2.6.1 Groups</td>
<td>68</td>
</tr>
<tr>
<td></td>
<td>2.6.2 Example of a Group</td>
<td>68</td>
</tr>
<tr>
<td></td>
<td>2.6.3 Representation of a Group</td>
<td>68</td>
</tr>
<tr>
<td></td>
<td>2.6.4 Green’s Function</td>
<td>71</td>
</tr>
<tr>
<td></td>
<td>2.6.5 Plane-Wave Representation of the Green’s Function</td>
<td>73</td>
</tr>
<tr>
<td>2.7</td>
<td>Conclusion</td>
<td>74</td>
</tr>
<tr>
<td></td>
<td>References</td>
<td>74</td>
</tr>
<tr>
<td>3</td>
<td>FMM and MLFMA in 3D and Fast Illinois Solver Code</td>
<td>77</td>
</tr>
<tr>
<td>3.1</td>
<td>Introduction</td>
<td>77</td>
</tr>
<tr>
<td>3.2</td>
<td>Three-Dimensional FMM and MLFMA</td>
<td>78</td>
</tr>
<tr>
<td></td>
<td>3.2.1 Integral Equations and the Method of Moments</td>
<td>78</td>
</tr>
<tr>
<td></td>
<td>3.2.2 Three-Dimensional FMM</td>
<td>80</td>
</tr>
<tr>
<td>3.3</td>
<td>Multilevel Fast Multipole Algorithm</td>
<td>83</td>
</tr>
<tr>
<td>3.4</td>
<td>Error Analysis in FMM and MLFMA</td>
<td>85</td>
</tr>
<tr>
<td></td>
<td>3.4.1 Truncation Error in Scalar Green’s Function</td>
<td>86</td>
</tr>
<tr>
<td></td>
<td>3.4.2 Truncation Error in the Vector Green’s Function</td>
<td>88</td>
</tr>
<tr>
<td></td>
<td>3.4.3 Error in Numerical Integral</td>
<td>92</td>
</tr>
<tr>
<td></td>
<td>3.4.4 Error in Local Interpolation</td>
<td>92</td>
</tr>
<tr>
<td>3.5</td>
<td>Large-Scale Computing</td>
<td>92</td>
</tr>
<tr>
<td></td>
<td>3.5.1 Block Diagonal Preconditioner</td>
<td>92</td>
</tr>
<tr>
<td></td>
<td>3.5.2 Initial Guess</td>
<td>93</td>
</tr>
<tr>
<td></td>
<td>3.5.3 Approximation of Bistatic RCS to Monostatic RCS</td>
<td>94</td>
</tr>
<tr>
<td></td>
<td>3.5.4 Interpolation of Translation Matrix in MLFMA</td>
<td>96</td>
</tr>
<tr>
<td></td>
<td>3.5.5 MLFMA for Calculating Radiation Fields</td>
<td>102</td>
</tr>
</tbody>
</table>
## Contents

3.6 Fast Illinois Solver Code (FISC) 103
   3.6.1 Capabilities 104
   3.6.2 Complexity and Accuracy 106
   3.6.3 Scaling of Memory Requirements 109
   3.6.4 CPU Time Scaling 110
   3.6.5 More Results and Summary 112

3.7 Conclusions 114

References 114

4 Parallelization of Multilevel Fast Multipole Algorithm on Distributed Memory Computers 119
   4.1 Introduction 119
   4.2 The MPI Programming Model 120
   4.3 Mathematical Preliminaries 122
      4.3.1 Notations 122
      4.3.2 Essentials of Diagonal Forms of Radiation Fields 122
      4.3.3 FMM Representation of Matrix Elements in Method of Moments 124
   4.4 The Parallel MLFMA 125
      4.4.1 The Algorithm 127
   4.5 Implementation Issues 132
      4.5.1 Storage of the Tree 132
      4.5.2 Construction of the Tree and Domain Decomposition 133
      4.5.3 Scaling of Translation Matrices 134
      4.5.4 A Costzone Scheme for Load Balancing 135
   4.6 ScaleME: A Brief Description 136
   4.7 Numerical Experiments 138
      4.7.1 The Integral Equation Formulation 138
      4.7.2 The TRIMOM+ScaleME Code 140
      4.7.3 Single Processor Performance 141
      4.7.4 Processor Scaling 142
      4.7.5 Large-Scale Problems 142
   4.8 ScaleME-2: An Improved Parallel MLFMA 144
   4.9 Conclusions 147

References 148
5 Multilevel Fast Multipole Algorithm at Very Low Frequencies 151
5.1 Introduction 151
5.2 Two-Dimensional Multilevel Fast Multipole Algorithm at Very Low Frequencies 153
  5.2.1 Core Equation of the 2D Undiagonalized Dynamic MLFMA 153
  5.2.2 Core Equation of the 2D Diagonalized Dynamic MLFMA 155
  5.2.3 2D Uniformly Normalized LF-MLFMA 156
  5.2.4 Nonuniformly Normalized Form of 2D LF-MLFMA 158
  5.2.5 Computational Complexity of 2D LF-MLFMA 158
  5.2.6 Applying 2D Dynamic MLFMA and LF-MLFMA to CFIE for PEC Structures 160
5.3 3D Multilevel Fast Multipole Algorithm at Very Low Frequencies 174
  5.3.1 General Formulations for the 3D Dynamic MLFMA 174
  5.3.2 Core Equation for 3D Diagonalized Dynamic MLFMA 176
  5.3.3 Core Equation for 3D LF-MLFMA 177
  5.3.4 Computational Complexity of 3D LF-MLFMA 180
  5.3.5 Core Equation for 3D Static MLFMA 181
  5.3.6 Rotation of the Translation Matrices for 3D LF-MLFMA and 3D Static MLFMA 184
  5.3.7 3D LF-MLFMA Based on RWG Basis 189
5.4 Conclusions 197
References 199

6 Error Analysis of Surface Integral Equation Methods 203
6.1 Introduction 203
  6.1.1 Surface Integral Equations and the Method of Moments 204
  6.1.2 Error Measures 206
  6.1.3 Approaches to Error Analysis 208
  6.1.4 Spectral Convergence Theory 211
6.2 Spectral Convergence Theory—2D 212
  6.2.1 Circular Cylinder—TM 213
  6.2.2 Circular Cylinder—TE 222
  6.2.3 Flat Strip—TM 225
  6.2.4 Flat Strip—TE 234
  6.2.5 Flat Strip—Edge Error 239
  6.2.6 Rectangular Cavity 242
  6.2.7 Higher-Order Basis Functions 247
## Contents

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.2.8 Summary</td>
<td>255</td>
</tr>
<tr>
<td>6.3 Spectral Convergence Theory—3D</td>
<td>256</td>
</tr>
<tr>
<td>6.3.1 Flat Plate</td>
<td>256</td>
</tr>
<tr>
<td>6.3.2 Rooftop Basis Functions</td>
<td>258</td>
</tr>
<tr>
<td>6.4 Iterative Solution Methods</td>
<td>261</td>
</tr>
<tr>
<td>6.4.1 Iteration Count Estimates</td>
<td>262</td>
</tr>
<tr>
<td>6.4.2 Condition Number Estimates</td>
<td>263</td>
</tr>
<tr>
<td>6.5 Conclusion</td>
<td>277</td>
</tr>
<tr>
<td>References</td>
<td>278</td>
</tr>
</tbody>
</table>

7 Advances in the Theory of Perfectly Matched Layers 283

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.1 Introduction</td>
<td>283</td>
</tr>
<tr>
<td>7.2 PML via Complex Space Coordinates</td>
<td>284</td>
</tr>
<tr>
<td>7.2.1 Frequency Domain Analysis</td>
<td>284</td>
</tr>
<tr>
<td>7.2.2 Time Domain Analysis</td>
<td>287</td>
</tr>
<tr>
<td>7.3 PML-FDTD for Dispersive Media with Conductive Loss</td>
<td>288</td>
</tr>
<tr>
<td>7.3.1 Time Domain Analysis</td>
<td>288</td>
</tr>
<tr>
<td>7.3.2 Dispersive Medium Models</td>
<td>288</td>
</tr>
<tr>
<td>7.3.3 Incorporation into FDTD Update</td>
<td>291</td>
</tr>
<tr>
<td>7.4 Maxwellian PML</td>
<td>293</td>
</tr>
<tr>
<td>7.5 Extension to (Bi)Anisotropic Media</td>
<td>294</td>
</tr>
<tr>
<td>7.5.1 Non-Maxwellian Formulation</td>
<td>295</td>
</tr>
<tr>
<td>7.5.2 Maxwellian Formulation</td>
<td>295</td>
</tr>
<tr>
<td>7.6 PML for Inhomogeneous Media</td>
<td>298</td>
</tr>
<tr>
<td>7.7 Curvilinear PML</td>
<td>299</td>
</tr>
<tr>
<td>7.7.1 Cylindrical PML-FDTD</td>
<td>299</td>
</tr>
<tr>
<td>7.7.2 Spherical PML-FDTD</td>
<td>305</td>
</tr>
<tr>
<td>7.7.3 Maxwellian PML in Cylindrical and Spherical Coordinates</td>
<td>307</td>
</tr>
<tr>
<td>7.7.4 Conformal (Doubly Curved) PML</td>
<td>310</td>
</tr>
<tr>
<td>7.8 Stability Issues</td>
<td>317</td>
</tr>
<tr>
<td>7.8.1 Cartesian PML Analysis</td>
<td>320</td>
</tr>
<tr>
<td>7.8.2 Cylindrical PML Analysis</td>
<td>323</td>
</tr>
<tr>
<td>7.8.3 Spherical PML Analysis</td>
<td>327</td>
</tr>
<tr>
<td>7.8.4 Imposing Stability a Posteriori: The Quasi-PML</td>
<td>330</td>
</tr>
<tr>
<td>7.9 Generalized PML-FDTD Schemes</td>
<td>331</td>
</tr>
<tr>
<td>7.9.1 Cylindrical PML-PLRC-FDTD: Split-Field Formulation</td>
<td>332</td>
</tr>
</tbody>
</table>
xii Fast and Efficient Algorithms in CEM

7.9.2 Cylindrical PML-PLRC-FDTD: Maxwellian Formulation 334
7.10 Unified Theory: Brief Discussion 336
  7.10.1 PML as a Change on the Metric of Space 336
  7.10.2 Metric and Topological Structure of Maxwell’s Equations 339
  7.10.3 Hybrid PMLs 341
References 342

8 Fast Forward and Inverse Methods for Buried Objects 347
  8.1 Introduction 347
  8.2 Green’s Functions 349
    8.2.1 Green’s Function in Integral Equation 350
    8.2.2 Green’s Function for Incident Field 351
    8.2.3 Green’s Function for Scattered Field 352
    8.2.4 Reduction to 2D Case 353
  8.3 Fast Forward Scattering Methods 354
    8.3.1 2D Buried Dielectric Cylinders 354
    8.3.2 3D Buried Conducting Plates 360
    8.3.3 3D Buried Dielectric Objects 364
  8.4 Detection of Buried Objects Using Forward Method 370
    8.4.1 VETEM System 370
    8.4.2 Numerical Modeling: Loop-Antenna Model 372
    8.4.3 Numerical Modeling: Magnetic-Dipole Model 374
    8.4.4 Simulation Results 375
  8.5 Fast Inverse Scattering Methods 380
    8.5.1 Single-Frequency DBIM Algorithm for 2D Objects 382
    8.5.2 Multifrequency DBIM Algorithm for 2D Objects 392
    8.5.3 DT Algorithm for 2D Objects 396
    8.5.4 DT Algorithm for 3D Objects 407
References 418

9 Low-Frequency Scattering from Penetrable Bodies 425
  9.1 Introduction 425
  9.2 Low-Frequency Scattering from a Single Penetrable Body 428
    9.2.1 A Brief Review of Basis Functions 428
    9.2.2 General Equations and Frequency Normalization 432
    9.2.3 Interpretations 436
<table>
<thead>
<tr>
<th>Chapter</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>9.3</td>
<td>Scattering from a Multibody</td>
<td>440</td>
</tr>
<tr>
<td>9.3.1</td>
<td>PMCHWT Formulation for Multibody Problem</td>
<td>440</td>
</tr>
<tr>
<td>9.3.2</td>
<td>Number-of-Unknowns Reduction Scheme for RWG Basis</td>
<td>443</td>
</tr>
<tr>
<td>9.3.3</td>
<td>Number-of-Unknowns Reduction Scheme for Loop-Tree Basis</td>
<td>449</td>
</tr>
<tr>
<td></td>
<td>References</td>
<td>458</td>
</tr>
<tr>
<td>10</td>
<td>Efficient Analysis of Waveguiding Structures</td>
<td>461</td>
</tr>
<tr>
<td>10.1</td>
<td>Introduction</td>
<td>461</td>
</tr>
<tr>
<td>10.2</td>
<td>Finite Difference Formulation</td>
<td>462</td>
</tr>
<tr>
<td>10.2.1</td>
<td>Boundary Conditions</td>
<td>465</td>
</tr>
<tr>
<td>10.3</td>
<td>Solution to the Sparse Matrix Equation</td>
<td>467</td>
</tr>
<tr>
<td>10.3.1</td>
<td>Complexity and Storage Issues</td>
<td>468</td>
</tr>
<tr>
<td>10.4</td>
<td>Waveguide Discontinuities</td>
<td>469</td>
</tr>
<tr>
<td>10.4.1</td>
<td>The Single Junction Problem</td>
<td>471</td>
</tr>
<tr>
<td>10.4.2</td>
<td>The n-Junction Problem</td>
<td>474</td>
</tr>
<tr>
<td>10.5</td>
<td>Numerical Examples</td>
<td>477</td>
</tr>
<tr>
<td>10.6</td>
<td>Conclusions</td>
<td>480</td>
</tr>
<tr>
<td></td>
<td>References</td>
<td>484</td>
</tr>
<tr>
<td>11</td>
<td>Volume-Surface Integral Equation</td>
<td>487</td>
</tr>
<tr>
<td>11.1</td>
<td>Introduction</td>
<td>487</td>
</tr>
<tr>
<td>11.2</td>
<td>The Formulation of the Integral Equations</td>
<td>488</td>
</tr>
<tr>
<td>11.2.1</td>
<td>Volume Integral Equation (VIE)</td>
<td>489</td>
</tr>
<tr>
<td>11.2.2</td>
<td>Hybrid Volume-Surface Integral Equation (VSIE)</td>
<td>491</td>
</tr>
<tr>
<td>11.3</td>
<td>Numerical Solution of the Hybrid VSIE</td>
<td>492</td>
</tr>
<tr>
<td>11.3.1</td>
<td>Mesh Generating</td>
<td>492</td>
</tr>
<tr>
<td>11.3.2</td>
<td>Discretization of the Hybrid Integral Equation</td>
<td>495</td>
</tr>
<tr>
<td>11.3.3</td>
<td>Mesh Termination</td>
<td>499</td>
</tr>
<tr>
<td>11.3.4</td>
<td>Enforcing the Continuity Condition</td>
<td>501</td>
</tr>
<tr>
<td>11.3.5</td>
<td>Other Cell Shapes</td>
<td>502</td>
</tr>
<tr>
<td>11.4</td>
<td>Combined Field Integral Equation</td>
<td>506</td>
</tr>
<tr>
<td>11.5</td>
<td>Singular Integral Treatments</td>
<td>508</td>
</tr>
<tr>
<td>11.6</td>
<td>Solution of VSIE by Fast Multipole Method</td>
<td>513</td>
</tr>
<tr>
<td>11.7</td>
<td>Numerical Examples</td>
<td>514</td>
</tr>
<tr>
<td>11.7.1</td>
<td>Results by Volume Integral Equation</td>
<td>515</td>
</tr>
</tbody>
</table>
# 11.7.2 Results by Hybrid Volume-Surface Integral Equation  
11.8 Other Applications  
11.8.1 Indoor Radio Wave Propagation Simulation  
11.8.2 Microwave Thermal Effect Simulation  
11.8.3 Antenna Radome Modeling  
References  

## 12 Finite Element Analysis of Complex Axisymmetric Problems  

### 12.1 Introduction  

### 12.2 Formulation  
12.2.1 Problem Definition  
12.2.2 Variational Formulation  
12.2.3 Solution of the Equations  
12.2.4 Far-Field Calculations  

### 12.3 Cylindrical PML  
12.3.1 Parameter Definitions  
12.3.2 Systematic Error Reduction  

### 12.4 Numerical Results  
12.4.1 Scattering  
12.4.2 Radiation  

### 12.5 BOR with Appendages  

### 12.6 Conclusion  
References  

## 13 Hybridization in Computational Electromagnetics  

### 13.1 Introduction  

### 13.2 Hybrid FEM/ABC Technique  
13.2.1 Problem Statement  
13.2.2 Finite Element Analysis  
13.2.3 Numerical Results  

### 13.3 Hybrid FEM/BIE Technique  
13.3.1 Formulation  
13.3.2 Application of MLFMA  
13.3.3 Numerical Results  

### 13.4 Hybrid FEM/AABC Technique  
13.4.1 Formulation  

References
Contents

13.4.2 Numerical Results 598
13.5 Hybrid FEM/SBR Technique 602
  13.5.1 Formulation 604
  13.5.2 Scattered Field Calculation 608
  13.5.3 Analysis of the Hybrid Technique 609
  13.5.4 Numerical Results 610
13.6 Hybrid MOM/SBR Technique 614
  13.6.1 Formulation 615
  13.6.2 Scattered Field Calculation 618
  13.6.3 Iterative Improvement 619
  13.6.4 Numerical Results 620
13.7 Summary 621
References 628

14 High-Order Methods in Computational Electromagnetics 637
  14.1 Introduction 637
  14.2 Higher-Order MOM and MLFMA 638
    14.2.1 Formulation 639
    14.2.2 Numerical Examples 645
  14.3 Point-Based Implementation of Higher-Order MLFMA 652
    14.3.1 Formulation 654
    14.3.2 Complexity Analysis 657
    14.3.3 Numerical Results 658
  14.4 Higher-Order FEM 658
    14.4.1 Higher-Order Tetrahedral Elements 661
    14.4.2 Application to Cavity Scattering 664
  14.5 Mixed-Order Prism Elements 672
  14.6 Point-Based Grid-Robust Higher-Order Bases 680
    14.6.1 Vector Basis Functions 682
    14.6.2 MOM Formulation 686
  14.7 Numerical Results 688
  14.8 Summary 693
References 694

15 Asymptotic Waveform Evaluation for Broadband Calculations 699
  15.1 Introduction 699
15.2 The AWE Method 700
15.3 Analysis of Metallic Antennas 702
  15.3.1 Formulation 703
  15.3.2 Numerical Examples 706
15.4 Analysis of Metallic Scatterers 708
  15.4.1 Formulation 708
  15.4.2 Numerical Examples 713
15.5 Analysis of Dielectric Scatterers 713
  15.5.1 Formulation 714
  15.5.2 Numerical Examples 717
15.6 Analysis of Microstrip Antennas 718
  15.6.1 Formulation 718
  15.6.2 Numerical Examples 720
15.7 Summary 725
References 725

16 Full-Wave Analysis of Multilayer Microstrip Problems 729
  16.1 Introduction 729
  16.2 Green’s Functions for Multilayer Media 730
  16.3 The Method-of-Moments Solution 737
  16.4 Fast Frequency-Sweep Calculation 746
  16.5 The Conjugate Gradient–FFT Method 752
  16.6 The Adaptive Integral Method 759
  16.7 The Multilevel Fast Multipole Algorithm 764
  16.8 Summary 770
References 772

17 The Steepest-Descent Fast Multipole Method 781
  17.1 Introduction 781
  17.2 Field Evaluation on Quasi-Planar Surfaces 782
    17.2.1 The Scalar Case 782
    17.2.2 The Vector Case 787
  17.3 Computational Complexity Estimates 788
  17.4 Scattering from Random Rough Surfaces 791
    17.4.1 Model Development 791
    17.4.2 Integral Equation Formulations 792
Contents

17.4.3 SDFMM-Based Solutions 793
17.4.4 Simulation Results 795

17.5 Quantum Well Grating Analysis 798
  17.5.1 Introduction and Formulation 798
  17.5.2 Periodic and Quasi-Random Grating Analysis 802
  17.5.3 Random Rough Surface Couplers 804

17.6 Analysis of Microstrip Antenna Arrays on Finite Substrates 804
  17.6.1 Introduction 804
  17.6.2 Integral Equation Formulation and SDFMM Solution 805
  17.6.3 MOM Formulation 807
  17.6.4 Simulation Results 808

17.7 Conclusion 810

References 810

18 Plane-Wave Time-Domain Algorithms 815
  18.1 Introduction 815
  18.2 The Marching-on-in-Time Method 817
  18.3 The Plane-Wave Time-Domain Algorithm 819
    18.3.1 Plane Wave Decomposition 820
    18.3.2 Implementation Issues 826
  18.4 Implementation of the PWTD-Enhanced MOT Schemes 830
    18.4.1 A Two-Level PWTD-Enhanced MOT Algorithm 831
    18.4.2 A Multilevel PWTD-Enhanced MOT Algorithm 836
  18.5 The Windowed Plane-Wave Time-Domain Algorithm 842
    18.5.1 Windowed Plane-Wave Decomposition 843
    18.5.2 Implementation Using Sampled Field Representations 844
  18.6 Implementation of the Windowed PWTD-Enhanced MOT Schemes 849
    18.6.1 Sphere-to-Sphere Translation 850
    18.6.2 A Two-Level Windowed PWTD-Enhanced MOT Algorithm 853
    18.6.3 A Multilevel Windowed PWTD-Enhanced MOT Algorithm 854
  18.7 Summary 855

References 856

19 Plane-Wave Time-Domain Algorithm Enhanced Time-Domain Integral
Equation Solvers 859
  19.1 Introduction 859
19.2  Formulation  
   19.2.1  Integral Equations  
   19.2.2  Marching-on-in-Time Formulation  
19.3  Plane-Wave Time-Domain Algorithm  
   19.3.1  Plane Wave Representations  
   19.3.2  Implementation of Two-Level PWTD Enhanced MOT Solvers  
   19.3.3  Complexity Analysis  
19.4  Numerical Results  
   19.4.1  Efficacy of the CFIE  
   19.4.2  Validating the PWTD-Augmented MOT Solver  
   19.4.3  Efficacy of the PWTD-Augmented MOT Scheme for Large-Scale Analysis  
19.5  Summary  
References  

About the Authors  

Index
Preface

This book documents recent advances in computational electromagnetics performed under the auspices of the Center for Computational Electromagnetics at the University of Illinois, funded mainly by the Multidisciplinary University Research Initiative (MURI), a program administered by the Air Force Office of Scientific Research. Other funding agencies also contributed to the success of the Center, such as the National Science Foundation, Office of Naval Research, Army Research Office, and Department of Energy.

There is a tremendous need to bring the science of electromagnetic simulation, also known as computational electromagnetics, to the same confidence level as that achieved by circuit simulation. However, computational electromagnetics involves solving Maxwell’s equations, which are more complex than circuit equations. It is hoped that one day electromagnetic simulation will master this complexity and enjoy the same pervasiveness in engineering design as does circuit simulation. We are grateful for the foresight of these funding agencies who share our passion for developing this technology.

This book does not pretend to be complete, as it reflects our viewpoint of computational electromagnetics. However, we believe that the knowledge required to support electromagnetic simulation in a sophisticated manner has to come from physicists, engineers, mathematicians, and computer scientists. Since electrical engineering is an offshoot of applied physics, we play the role of applied physicists in the development of this technology: we develop this technology based on our physical insight into the problems, while drawing on knowledge from mathematicians and computer scientists. The presentation style of most of the chapters of this book is in the manner of applied physicists or of traditional electromagneticists—hopefully, we sacrifice mathematical rigor for physical clarity.
This book is not an introduction to computational electromagnetics. It documents recent advances in computational electromagnetics in the manner of a monograph. A seasoned researcher in the area of computational electromagnetics should have little difficulty reading the material. It is also hoped that a graduate student or a professional with some preliminary background in computational electromagnetics or a classicist in electromagnetics who has done some rapid background reading, can easily digest the work reported in this book. For one who intends to perform research in this area, this book will be an excellent starting point. The variety of topics covered is sufficient to nourish many different research directions in this very interesting field.

Even though this book deals only with linear problems associated with Maxwell’s equations, it can be gleaned from a cursory reading that such problems are rich; they are amenable to different mathematical analyses, and allow for different and interesting algorithm designs. Because of the linearity of the problems, both differential equation and integral equation solvers can be developed. Moreover, the problems can be solved in the frequency domain as well as the time domain, enhancing the efficiency and enriching the variety of these methods.

Solutions to Maxwell’s equations have been sought since the very early days of the equations’ discovery. Electromagnetic analysis has always played an important role in understanding many scientific and engineering problems.

Chapter 1 gives an introduction to electromagnetic analysis and explains how the field has evolved into computational electromagnetics in the last few decades. It also introduces, in a very simplified manner, the recent fast algorithms developed to solve Maxwell’s equations. The chapter also attempts to give a historical perspective on electromagnetic analysis and to describe how far we have come since the advent of Maxwell’s equations.

Chapter 2 presents an introduction to the fast multipole method (FMM) and the multilevel fast multipole algorithm (MLFMA) in two dimensions. Interpolation, truncation, and integration errors are discussed. An attempt is also made to relate FMM to group theory, and to the inherent symmetry of space.

Chapter 3 describes the three-dimensional version of FMM and MLFMA and demonstrates the application of the fast algorithm to real-world problems. The algorithm has also been parallelized on a shared-memory machine, and tour-de-force computation involving close to 10 million unknowns is the most important achievement of this work.

Chapter 4 outlines the distributed-memory parallelization of MLFMA, encapsulated in a code called ScaleME (Scaleable Multipole Engine). The parallelization of MLFMA on a distributed memory machine is not an easy task, because different parts of the computation may reside on different processors. The increased communication cost with more processors can be an issue here. A 10-million-unknown problem has also been solved with ScaleME.
Chapter 5 reports on the low-frequency solution of Maxwell’s equations using fast algorithms. This chapter describes the treatment needed for FMM and MLFMA to prevent their catastrophic breakdown at low frequencies. It also describes a method to apply the LF-MLFMA based on Rao-Willon-Glisson (RWG), wire, and wire-surface bases while the intrinsic expansion bases are still the loop-tree-star bases. These bases are designed for low-frequency problems to make the LF-MLFMA efficient for problems with global loops.

Chapter 6 delves into different error issues involved when solving surface integral equations related to Maxwell’s theory. Discretization error due to the use of basis functions, and integration error by replacing integrals with summation are discussed. Errors result from solving the matrix equation, and deconditioning of the matrix equation by MOM and its impact on errors are studied. This chapter also discusses deconditioning due to the near-resonance problem and the low-frequency breakdown problem.

Chapter 7 deals with a recent topic of intense interest in differential equation solvers—the theory of perfectly matched layers (PML). The concept of complex coordinate stretching is discussed. PML is generalized to curvilinear coordinates as well as to complex media. In this chapter, stability issues related to PML are studied, and a unified analysis of various PML formulations using differential forms is included.

Chapter 8 addresses the issue of efficiently solving the forward and inverse problems for buried objects using FFT-based methods. The detection of buried objects usually involves loop antennas, and the forward problem involving the solution of loop antennas over a buried object is discussed in great detail. Moreover, recent advances in different inversion algorithms are also described.

Chapter 9 touches upon solving the penetrable problem at very low frequencies. The low-frequency problem encountered in Chapter 5 for metallic objects also occurs for dielectric and lossy material objects. This chapter describes a way to solve this problem so that the solution of integral equations remains stable all the way from zero frequency to microwave frequencies.

Chapter 10 describes an algorithm to solve three-dimensional waveguide structures using numerical mode matching, but using the finite difference method. The spectral Lanczos decomposition method is used to find the modes. An algorithm with $O(N)$ memory complexity and $O(N^{1.5})$ computational complexity is achieved.

Chapter 11 addresses the problem of solving the volume integral equation concurrently with the surface integral equation. This is particularly important when dealing with structures having metals as well as dielectric materials. The solutions are also accelerated with MLFMA as demonstrated in the chapter. Many practical illustrations of the use of this solution technique are given in this chapter.

Chapter 12 deals with solving axially symmetric, body-of-revolution (BOR) geometry using the finite element method (FEM). This reduces a three-dimensional problem to two dimensions, greatly enhancing the efficiency of the solution. Both
material-coated and metallic objects are considered. The chapter also shows the practical use of cylindrical PML for truncating the FEM mesh. Treatment of BOR geometry with appendages is also considered.

Chapter 13 reports on the hybridization in computational electromagnetics. Hybridization between FEM and the absorbing boundary condition (ABC) is discussed alongside the boundary integral equation (BIE), MLFMA, adaptive absorbing boundary condition (AABC), and shooting and bouncing ray (SBR). Hybridization between MOM and SBR is also considered. AABC is a promising method of hybridizing FEM with fast solvers in the future.

Chapter 14 presents different higher-order methods in computational electromagnetics. Higher-order methods for the surface integral equation as well as for FEM are considered. Also, the efficient coupling of higher-order methods to fast solvers such as MLFMA is discussed. In particular, the use of point-based MLFMA is illustrated. Moreover, a higher-order grid-robust method is also studied in this chapter.

Chapter 15 touches on the topic of asymptotic waveform evaluation (AWE) for broadband calculation in electromagnetics. Illustrations of this acceleration technique for broadband calculation are given for metallic antennas, wire antennas, dielectric scatterers, and microstrip antennas.

Chapter 16 details the analysis of microstrip structure on top of a layered medium. The derivation of the layered medium Green’s function together with its numerical approximation by the complex images is discussed. The use of the fast frequency sweep method, adaptive integral method, and MLFMA to accelerate solution speed is studied. A higher-order method to improve solution accuracy is also demonstrated.

Chapter 17 reviews the steepest-descent FMM (SDFMM) to accelerate the solution speed of quasi-planar structures. For this class of structures, this method reduces both the computational and memory complexity of MLFMA from $O(N \log N)$ to $O(N)$. Applications to scattering from random rough surfaces, quantum-well gratings, and microstrip antennas are demonstrated with this analysis method.

Chapter 18 elaborates on the plane-wave time-domain (PWTD) algorithm, which is an ingenious way of arriving at the time-domain equivalent of FMM and MLFMA. The integral equation is solved using the marching-on-in-time (MOT) method. Stability and accuracy issues are carefully analyzed in this chapter. Both the two-level and multilevel algorithms are presented and demonstrated with examples.

Chapter 19 further develops PWTD for large-scale and real-world applications. The use of PWTD with the magnetic field integral equation (MFIE), electric field integral equation (EFIE), and combined field integral equation (CFIE) is illustrated. Furthermore, scattering and error analysis from complex targets such as aircraft, almond shapes, and cone-spheres are considered.

Even though a large variety of topics is covered here, we do feel that there is still a myriad of problems in computational electromagnetics begging to be solved. Due to the complex nature of computational electromagnetics compared to circuit simulation, the robustness and stability of these algorithms are still issues to be addressed.
Another issue is the computational labor associated with these algorithms—more research needs to be done to enhance their speed. We hope, however, that the work at our Center marks a new beginning in the era of fast algorithms in computational electromagnetics.

During the MURI support, we have demonstrated our ability to solve problems involving 10 million unknowns using the supercomputing facilities of the University of Illinois. With continued support in this field, together with improvements in computer technology, we predict that a decade from now, solving a problem of this size will be routine for many applications.

*If only electromagnetic fields can talk, they will speak volumes!*

WENG CHO CHEW

*Urbana-Champaign, Illinois, June 2001*
Acknowledgments

First, we are indebted to many colleagues who have contributed to this field. We are also indebted to many from whom we have learned this material, making this work possible. We also owe much to many graduate students, postdoctoral associates, and research scientists who have worked tirelessly to make this work possible. Many of them are coauthors of chapters in this book.

Financial support from the following organizations in the course of our research is gratefully acknowledged:

- Air Force Office of Scientific Research under MURI grant F49620-96-1-0025;
- National Science Foundation;
- Office of Naval Research;
- Department of Energy.

In addition, a number of industrial organizations have contributed to our research— notably, HRL, Intel, Lockheed-Martin, Mobil-Exxon, MRC, Northrop-Grumman, Raytheon, and Schlumberger. At the University of Illinois, we thank NCSA for the use of the supercomputing facilities and support from the CSE Program. One of the authors of Chapter 19, AAE, would like to thank Gebze Institute of Technology for their support.

We thank Jamie Hutchinson of our Publications Office, who painstakingly read all chapters and checked for editorial corrections. For their willingness to serve and provide feedback during the course of this program, many thanks also go to the advisory board members of our Center: Tom Blalock, Bill Hall, Kristopher Kim, Charlie Liang, Bob Mailloux, Louis Medgyesi-Mitschang, Don Pflug, Maurice
Sancer, Joe Shang, Anthony Terzuoli, Steve Wandzura, Arthur Yaghjian, and Long Yu. We owe Dennis Andersh and S. W. Lee of SAIC (formerly of UIUC) a word of gratitude for their moral support and interest during this program. Finally, we thank Arje Nachman of AFOSR, whose no-nonsense approach to monitoring our program provides constant impetus.

Weng Cho Chew thanks several students—Alaeddin Aydiner, Yunhui Chu, Eric Forgy, Larkin Hastriter, and Lijun Jiang—for proofreading and providing useful feedback on some of the chapters. Thanks is also due to Sanjay Velamparambil, who spent hours perfecting the template to meet the requirements of the publisher. Last but not least, we are grateful to our wives for their support despite the trying times they went through during the course of this research.

Weng Cho Chew, Jian-Ming Jin, Eric Michielssen, and Jiming Song
About the Authors

Kemal Aygun received his B.S. degree from the Middle East Technical University, Ankara, Turkey, in 1995, and his M.S. degree from the University of Illinois at Urbana-Champaign, in 1997, both in electrical engineering. He is currently pursuing his Ph.D. degree at UIUC. He also received the 1999 and 2000 Computational Science and Engineering Fellowship. His e-mail address is aygun@decwa.ece.uiuc.edu.

Siyuan Chen is a manufacturing development engineer in the Lightwave Division of Agilent Technologies, Inc. He received his Ph.D. degree in 2000 from the University of Illinois at Urbana-Champaign. His current interests include computational electromagnetics and microwave measurement system design. His e-mail address is siyuan.chen@agilent.com.

Weng Cho Chew is Founder Professor, College of Engineering, and a professor of electrical and computer engineering, University of Illinois at Urbana-Champaign. He is also the director of the Center for Computational Electromagnetics and the Electromagnetics Laboratory. He earned his B.S., M.S., and Ph.D. degrees from the Massachusetts Institute of Technology in 1976, 1978, and 1980, respectively. He has worked as a department manager at Schlumberger-Doll Research and joined the faculty at the University of Illinois in 1985. He is interested in fast algorithms for scattering and inverse scattering computations and electromagnetics for various applications. In addition, he has written a book entitled Waves and Fields in Inhomogeneous Media and published over 230 journal papers and over 300 conference papers. He is an IEEE Fellow and was an NSF Presidential Young Investigator. He received the 2000 IEEE Graduate Teaching Award and the 2001 UIUC Campus Award for Excellence in Graduate and Professional Teaching. He coauthored a paper that won the Schelkunoff Prize Paper award in 2001. His e-mail address is w-chew@uiuc.edu.

Tie Jun Cui is a research scientist at the Center for Computational Electromagnetics, Department of Electrical and Computer Engineering, University of Illinois at Urbana-Champaign. He received his B.S., M.S., and Ph.D. degrees in electrical engineering from Xidian University in 1987, 1990, and 1993, respectively. His research interests include wave propagation, scattering, inverse scattering, landmine detection, geophysical subsurface sensing, fast algorithms, and intergrated circuit simulations. His e-mail address is tiecui@uiuc.edu.

Kalyan Donepudi is a graduate student in the Department of Electrical and Computer Engineering at the University of Illinois at Urbana-Champaign. His research
interests are fast methods, high-frequency scattering, and higher-order solutions. His e-mail address is donepudi@students.uiuc.edu.

A. Arif Ergin is an assistant professor at the Gebze Institute of Technology, Kocaeli, Turkey. He received his Ph.D. degree from the University of Illinois at Urbana-Champaign in 2000. His current research interests include computational methods for analyzing wave propagation and scattering in acoustics and electromagnetics, antenna design and measurements, and electromagnetic compatibility. His e-mail address is aergin@penta.gyte.edu.tr.

Andrew D. Greenwood received his B.S. (summa cum laude) and M.S. degrees in electrical engineering from Brigham Young University, Provo, Utah, in 1993 and 1995, respectively. He received his Ph.D. degree in electrical engineering from the University of Illinois at Urbana-Champaign in 1998. He received the Air Force Palace Knight Fellowship and is an IEEE member. He is currently with the Air Force Research Laboratory, Directed Energy Directorate, Kirtland AFB, New Mexico. His research interest is numerical techniques for electromagnetic problems. His e-mail address is Andrew.Greenwood@kirtland.af.mil.

Vikram Jandhyala is an assistant professor in the Department of Electrical Engineering, University of Washington, Seattle, Washington. He received his Ph.D. degree from the University of Illinois at Urbana-Champaign in 1998 and worked as a research and development engineer developing fast electromagnetic solvers at Ansoft, Pittsburgh, from 1998 to 2000. His research interests include computational electromagnetics, integral equations, fast algorithms, and field solvers for high-speed circuits. He is a recipient of the NSF Career Award (2001), Outstanding Graduate Research Award at UIUC (1998), and IEEE Microwave Graduate Fellowship (1997). His e-mail address is jandhyala@ee.washington.edu.

Dan Jiao is currently working toward a Ph.D. degree in electrical engineering at the University of Illinois at Urbana-Champaign. She received the 2000 Raj Mittra Outstanding Research Award presented by the Department of Electrical and Computer Engineering, University of Illinois at Urbana-Champaign. She has published over 20 papers in refereed journals. Her current research interests include fast computational methods in electromagnetics and time-domain numerical techniques. Her e-mail address is danjiao@uiuc.edu.

Jian-Ming Jin is a professor of electrical and computer engineering and an associate director of the Center for Computational Electromagnetics at the University of Illinois at Urbana-Champaign. He has authored *The Finite Element Method in Electromag-
netics and Electromagnetic Analysis and Design in Magnetic Resonance Imaging, and coauthored Computation of Special Functions. He has served as an associate editor of Radio Science and IEEE Transactions on Antennas and Propagation. He received the 1994 National Science Foundation Young Investigator Award and the 1995 Office of Naval Research Young Investigator Award. He also received the 1997 and 2000 Xerox Research Awards from the University of Illinois and was appointed the first Henry Magnuski Outstanding Young Scholar in the Department of Electrical and Computer Engineering in 1998. He was a Distinguished Visiting Professor in the Air Force Research Laboratory in 1999 and was elected an IEEE Fellow in 2000. His e-mail address is j-jin1@uiuc.edu.

Gang Kang received his Ph.D. degree from Peking University, Beijing, China, in 1998. From June 1998 to December 1999, he was a research associate in the Center for Computational Electromagnetics, University of Illinois at Urbana-Champaign. He is currently a research associate in the Department of Electrical Engineering, University of Utah, Salt Lake City, Utah. His interests include numerical techniques for electromagnetics and their applications in bioelectromagnetics, scattering, and antenna design. His e-mail address is gkang@ee.utah.edu.

Feng Ling is a senior engineer/scientist at the Semiconductor Products Section, Motorola, Inc., Tempe, Arizona. He received his B.S. and M.S. degrees in electrical engineering from Nanjing University of Science and Technology in 1993 and 1996, respectively, and the Ph.D. degree in electrical engineering from the University of Illinois at Urbana-Champaign in 2000. His research interests include computational electromagnetics and modeling of integrated passive components and interconnects for RF applications. His e-mail address is fengling@ieee.org.

Jian Liu received his B.S. and M.S. degrees in electrical engineering from the University of Science and Technology of China in 1995 and 1998, respectively. Since 1998, he has been working in the Center of Computational Electromagnetics at the University of Illinois at Urbana-Champaign as a research assistant. His research interests include the application of finite element and moment methods to electromagnetics problems. His e-mail address is jianliu@uiuc.edu.

Cai-Cheng Lu is an assistant professor in the Department of Electrical and Computer Engineering at the University of Kentucky. He received his Ph.D. degree in 1995 from the Department of Electrical and Computer Engineering, University of Illinois at Urbana-Champaign. His research interests are wave scattering, microwave circuit simulation, and antenna analysis. He is especially experienced in fast algorithms for computational electromagnetics. He is a recipient of the NSF Career Award as well as the ONR Young Investigator Award. His e-mail address is cclu@engr.uky.edu.
Eric Michielssen is an associate professor in the Department of Electrical and Computer Engineering, University of Illinois at Urbana-Champaign, and an associate director of the Center for Computational Electromagnetics. He received his Ph.D. degree from the University of Illinois at Urbana-Champaign in 1992 and has been on the UIUC faculty ever since. His research interests include all aspects of theoretical and applied computational electromagnetics, and specifically fast algorithms for solving frequency/time domain integral equations and robust electromagnetic synthesis techniques. He received the NSF Career Award in 1995, Applied Computational Electromagnetics Valued Service Award in 1998, the International Union of Radio Science Henry G. Booker Fellowship in 1999, Issac Koga Gold Medal in 1999, and Xerox Research Award from UIUC in 2001. His e-mail address is emichiel@uiuc.edu.

Kaladhar Radhakrishnan is a senior design engineer for Intel Corporation. He received his Ph.D. degree in 1999 from the University of Illinois at Urbana-Champaign. As part of his doctoral work, he developed efficient numerical algorithms for the analysis of waveguiding structures. His current area of work involves designing microprocessor packages to meet power delivery and signal integrity requirements.

Balasubramaniam Shanker is an assistant professor in the Department of Electrical and Computer Engineering at Iowa State University. He received his B.Tech degree from the Indian Institute of Technology, Madras, India, in 1989, and M.S. and Ph.D. degrees from Pennsylvania State University in 1992 and 1993, respectively. His research interests include all aspects of computational electromagnetics with an emphasis on fast time and frequency domain algorithms. His e-mail address is shanker@iastate.edu.

Jiming Song is a staff engineer/scientist in the Digital DNA Research Laboratory of the Semiconductor Products Sector of Motorola in Tempe, Arizona. He received his B.S. and M.S. degrees in physics from Nanjing University in China, and his Ph.D. degree in electrical engineering from Michigan State University. From 1993 to 2000, he worked as a postdoctoral research associate, research scientist, and visiting assistant professor at the University of Illinois at Urbana-Champaign. From 1996 to 2000, he also worked as a research scientist at SAIC-DEMACO. His research interests include modeling and simulation of interconnect and passive components, wave scattering using fast algorithms, wave interaction with inhomogeneous media, electromagnetic simulations for foliage penetration applications, and transient electromagnetic field. His e-mail address is Jiming.Song@motorola.com.
Fernando L. Teixeira is an assistant professor in the Department of Electrical Engineering at The Ohio State University, where he teaches courses in electromagnetics and wireless propagation. He is also affiliated with the ElectroScience Laboratory at Ohio State. He received his Ph.D. degree from the University of Illinois at Urbana-Champaign and did postdoctoral work at the Massachusetts Institute of Technology. His current research interests include computational electromagnetics, electromagnetic wave interaction with complex materials, and scattering.

Sanjay Velamparambil is a research scientist at the Center for Computational Electromagnetics, Department of Electrical and Computer Engineering, University of Illinois, Urbana-Champaign. He obtained his M. Eng. and Ph.D. degrees from the Indian Institute of Science, Bangalore. His research interests are fast algorithms and parallel computing. His e-mail address is sanjay@sunchew.ece.uiuc.edu.

Karl Warnick is an assistant professor in the Department of Electrical and Computer Engineering at Brigham Young University, Provo, Utah. He received the B.S. degree in 1994 and his Ph.D. degree in 1997, both from Brigham Young. His research interests are electromagnetics and numerical simulation methods. His e-mail address is warnick@ee.byu.edu.

Junsheng Zhao received his B.S. degree from Shandong University, China, in 1985, his M. Eng. degree from the Second Academy of the Ministry of the Astronautics Industry of China (now China Aerospace Corporation), in 1988, and his Ph.D. degree from Tsinghua University, China, in 1995, all in electrical engineering. He is currently a research scientist at the Department of Electrical and Computer Engineering, University of Illinois at Urbana-Champaign. His research interests include fast algorithms for computational electromagnetics, microwave integrated circuits, and ferrite devices. He coauthored a paper that won the Schelkunoff Prize Paper award in 2001. His e-mail address is zhao@sunchew.ece.uiuc.edu.