
The field or wave in a transmission line is TEM (Transmission Electromagnetic) because both the $\mathbf{H}$-field and the $\mathbf{E}$-field are transverse to the direction of propagation. If the wave is propagating in the $\hat{z}$-direction, then both $E_z$ and $H_z$ are zero for such a wave. In such a case, the fields are

$$\mathbf{E} = E_s, \mathbf{H} = H_s,$$

(1)

where we have used the subscript $s$ to denote fields transverse to the direction of propagation. We can also define a del operation such that

$$\nabla = \nabla_s + \hat{z}\frac{\partial}{\partial z},$$

(2)

where $\nabla_s$ is transverse to the $\hat{z}$-direction, and in Cartesian coordinate, it is $\nabla_s = \hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y}$. From

$$\nabla \times \mathbf{H} = \epsilon \frac{\partial \mathbf{E}}{\partial t},$$

(3)

or

$$\left(\nabla_s + \hat{z}\frac{\partial}{\partial z}\right) \times \mathbf{H}_s = \epsilon \frac{\partial \mathbf{E}_s}{\partial t}. \quad (4)$$

Since $\nabla \times \mathbf{H}_s$ points in the $\hat{z}$-direction, $\hat{z}\frac{\partial}{\partial z} \times \mathbf{H}_s$ is $\hat{z}$-directed, we have

$$\nabla_s \times \mathbf{H}_s = 0,$$ \hspace{1cm} (5)

$$\frac{\partial}{\partial z} (\hat{z} \times \mathbf{H}_s) = \epsilon \frac{\partial \mathbf{E}_s}{\partial t}. \hspace{1cm} (6)$$

Similarly, from $\nabla_s \times \mathbf{E}_s = -\mu \frac{\partial \mathbf{H}_s}{\partial t}$, we can show that

$$\nabla_s \times \mathbf{E}_s = 0,$$ \hspace{1cm} (7)

$$\frac{\partial}{\partial z} (\hat{z} \times \mathbf{E}_s) = -\mu \frac{\partial \mathbf{H}_s}{\partial t}. \hspace{1cm} (8)$$

Equations (5) and (7) shows that the transverse curl of the fields are zero. This implies that the fields in the transverse directions of a transmission line resembles that of the electrostatic fields. Furthermore, Equations (6) and (8) couple the $\mathbf{E}_s$ and $\mathbf{H}_s$ fields together. These two equations are the electromagnetic field analogues of the telegrapher’s equations.
A current in a coaxial cable will produce a magnetic field polarized in the $\phi$ direction. From Ampere’s Law, we have

$$\oint_C \mathbf{H_s} \cdot d\mathbf{l} = \int_A \mathbf{J} \cdot ds = I, \quad (9)$$

or

$$\int_0^{2\pi} \rho \, d\phi H_\phi = I. \quad (10)$$

Hence,

$$H_\phi(\rho, z, t) = \frac{I(z, t)}{2\pi \rho}. \quad (11)$$

If we assume that the inner conductor in the coaxial line is charged up with the line charge $Q$ in coulomb/m, then from $\oint \mathbf{E} \cdot \mathbf{n} \, ds = Q$, we have

$$2\pi \rho \varepsilon E_\rho = Q, \quad (12)$$

or

$$E_\rho = \frac{Q}{2\pi \rho \varepsilon}. \quad (13)$$

Since the potential between $a$ and $b$ is $\int_a^b E_\rho \, d\rho$, we have

$$V = \int_a^b E_\rho \, d\rho = \frac{Q}{2\pi \varepsilon} \ln \left(\frac{b}{a}\right). \quad (14)$$

Hence,

$$E_\rho(\rho, z, t) = \frac{V(z, t)}{\rho \ln \left(\frac{b}{a}\right)} = \frac{Q(z, t)}{2\pi \varepsilon \rho}. \quad (15)$$

The ratio $\frac{Q}{V}$ is the capacitance per unit length, and it is

$$C = \frac{2\pi \varepsilon}{\ln \left(\frac{b}{a}\right)}. \quad (16)$$
If \( \mathbf{E}_s = \rho \mathbf{E}_\rho, \mathbf{H}_s = \phi \mathbf{H}_\phi \), equations (6) and (8) become

\[
\begin{align*}
\frac{\partial}{\partial z} H_\phi &= -\varepsilon \frac{\partial E_\rho}{\partial t}, \\
\frac{\partial}{\partial z} E_\rho &= -\mu \frac{\partial H_\phi}{\partial t}.
\end{align*}
\]

(17) \hspace{1cm} (18)

Substituting (11) for \( H_\phi \) and (15) for \( E_\rho \), we get

\[
\frac{\partial}{\partial z} I(z, t) = - \frac{2\pi \varepsilon}{\ln \left( \frac{b}{a} \right)} \frac{\partial V}{\partial t},
\]

(19)

and

\[
\frac{\partial}{\partial z} V(z, t) = - \frac{\mu \ln \left( \frac{b}{a} \right)}{2\pi} \frac{\partial I}{\partial t}.
\]

(20)

This is just the telegrapher’s equations derived from Maxwell’s equations. \( C \) is given by (16) while the inductance per unit length \( L \) is obtained by comparing (20) with the telegrapher’s equations

\[
L = \frac{\mu \ln \left( \frac{b}{a} \right)}{2\pi}.
\]

(21)

Note that the velocity of the wave on a transmission line is

\[
v = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{\mu \varepsilon}},
\]

(22)

which is independent of the dimensions of the line. This is because all TEM waves have velocity given by \( \frac{1}{\sqrt{\mu \varepsilon}} \).