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ECE 350 Lecture Notes

7. The Smith Chart

We have seen from Equation (6.9) that a generalized impedance can be defined as

$$Z(z) = \frac{\tilde{V}(z)}{\tilde{I}(z)} = Z_0 \frac{e^{-j\beta z} + \rho_v e^{+j\beta z}}{e^{-j\beta z} - \rho_v e^{+j\beta z}}. \tag{1}$$

The above can be written as

$$Z(z) = Z_0 \frac{1 + \rho_v e^{2j\beta z}}{1 - \rho_v e^{2j\beta z}} = Z_0 \frac{1 + \Gamma(z)}{1 - \Gamma(z)}, \tag{2}$$

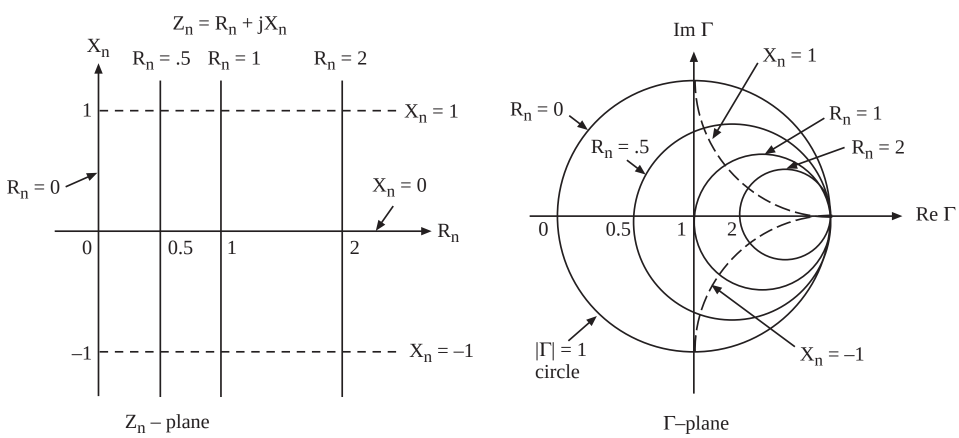
where $\Gamma(z)$ is as defined in (6.16). When $z = 0$, $Z(0) = Z_L$, and $\Gamma(0) = \rho_v$, and (2) becomes (6.25). Hence (6.25) is a special case of (2). We can introduce a ***normalized*** generalized impedance to be

$$Z_n(z) = \frac{Z(z)}{Z_0} = \frac{1 + \Gamma(z)}{1 - \Gamma(z)}. \tag{3}$$

Similarly,

$$\Gamma(z) = \frac{Z_n(z) - 1}{Z_n(z) + 1}. \tag{4}$$

Given $\Gamma(z)$, we can solve for $Z_n(z)$ in (3), and given $Z_n(z)$, we can solve for $\Gamma(z)$ in (4). It turns out that the mapping of $Z_n(z)$ to $\Gamma(z)$ and the mapping of $\Gamma(z)$ to $Z_n(z)$ are one-to-one. We shall next discuss a graphical method to solve (3) and (4) rapidly using the ***Smith Chart.***



Z_n is a complex number and can be represented by a point on the Z_n-plane, and Γ is a complex number and can be represented by a point on the complex Γ plane.

We noted that from Equation (4) that:

(i) When $Z_n = 0, \quad \Gamma = -1$.
(ii) When $Z_n = 1$, or $R_n = 1, X_n = 0, \quad \Gamma = 0$.
(iii) When $Z_n \to \infty$ in any direction, $\quad \Gamma \to 1$.
(iv) When $Z_n = jX_n, \quad |\Gamma| = 1$.
(v) When $Z_n = j$, or $R_n = 0, X_n = 1, \quad \Gamma = j$.
(vi) When $Z_n = -j$, or $R_n = 0, X_n = -1, \quad \Gamma = -j$.

If one works out the mapping from Z_n-plane to Γ-plane completely, one finds that the $R_n = 0$ line on Z_n-plane maps onto the unit-circle on the Γ-plane. Furthermore, the other $R_n = constant$ lines map into circles as shown. The $X_n = constant$ lines map into arcs like the $X_n \pm 1$ lines as shown. Hence, if one puts grids on the Γ-plane, one can read off the R_n and X_n associated with the corresponding Γ immediately, and, given the value of Γ, one can read off the values of R_n and X_n immediately.

The mappings (3) and (4) are known as bilinear transforms. A bilinear transform always maps a circle onto a circle.

Properties of a Smith Chart

(i) The normalized admittance $Y_n = 1/Z_n$, or the reciprocal of Z_n, can be found easily from a Smith Chart, because

$$\Gamma = \frac{Z_n - 1}{Z_n + 1} = \frac{1 - \frac{1}{Z_n}}{1 + \frac{1}{Z_n}} = \frac{1 - Y_n}{1 + Y_n} = -\frac{Y_n - 1}{Y_n + 1}. \tag{5}$$

(ii) The change of impedance along the line is obtained by adding or subtracting phase to $\Gamma(z)$ via the relationship

$$\Gamma(z) = \rho_v e^{2j\beta z}. \tag{6}$$

(iii)

$$\text{VSWR} = \frac{1 + |\rho_v|}{1 - |\rho_v|} = R_{n\,max}, \tag{7}$$

since the Smith Chart is a graphical tool to solve Equation (7), and $|\rho_v|$ is real, corresponding to a number on the $X_n = 0$ line. Notice that $1 <$ VSWR $< \infty$ always.