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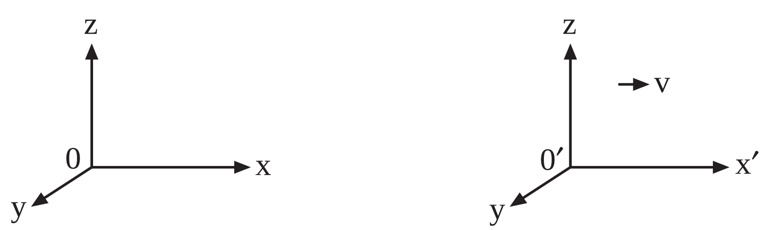
ECE 350 Lecture Notes

Doppler Effect

When we are in a Chicago subway station, notice that the pitch of the screaming express train is always higher when it is approaching you compared to when it is leaving you. This phenomenon is the result of the Doppler effect. It occurs in all waves, including electromagnetic waves. Because we live in an expanding universe, the light wave from stars in a distant galaxy appears to have a lower frequency than its actual frequency. The radio signal from an aircraft appears to be lower or higher than its actual frequency depending on if it is approaching or leaving the receiver location.

To understand Doppler effect in electromagnetics, we have to understand how the law of physics would change in different reference frames which are moving at different velocities. Since the reference frames are moving with respect to each other, one has to draw upon the theory of special relativity to fully understand Doppler effect. However, the theory of special relativity is beyond the scope of this course, and we will just use the simple theory of relativity, which is a good theory if the reference frames are moving slowly (compared to the speed of light) with respect to each other.



Let us assume that we have a plane wave described by

$$\Phi(z,t) = A\cos(\omega t - \beta z), \tag{1}$$

where $\beta = \omega/c$. This describes a wave moving at the velocity c in the (z,t) frame. Let us assume that there is (z',t) frame which is moving in the positive z direction whith a velocity v.

We assume that at $t = 0$, $z = z'$, i.e., the two frames coincide with each other. Then,

$$z' = z - vt \qquad or \qquad z = z' + vt. \tag{2}$$

To an observer in the moving frame, Equation (1) becomes

$$\begin{aligned}\Phi(z',t) &= A\cos(\omega t - \beta z' - \beta vt)\\ &= A\cos[\omega(1-\frac{v}{c})t - \beta z']\\ &= A\cos(\omega' t - \beta z), \end{aligned}\tag{3}$$

where

$$\omega' = \omega(1 - \frac{v}{c}). \tag{4}$$

Therefore, the frequency of the wave appears lower to the moving observer who is moving in the same direction as the travelling wave. If the observer is travelling in the opposite direction to the wave, then, the observer notices a rise in frequency of the wave.

The above derivation is non-relativistic, and hence is valid if $v/c \ll 1$. For larger v, a relativistic derivation is required.

For a wave traveling in a $\hat{c}$ direction, and an observer travelling in the with a velocity $\mathbf{v}$, the Doppler shift is given by the formula

$$\Delta\omega = -\omega\frac{\mathbf{v}\cdot\hat{c}}{c}, \tag{5a}$$

or

$$\Delta f = -f\frac{\mathbf{v}\cdot\hat{c}}{c}. \tag{5b}$$

The Doppler effect is also used by the weather radar to detect wind velocity and by the police radar to detect the velocity of moving vehicle. Since for most earthly velocities, the ratio v/c is a tiny ratio, Δf is a small number unless the frequency of the wave or radar f is huge. Therefore, most Doppler radars work in the microwave regime where the carrier frequency is around GHz.