A Frequency-Hopping Approach for Microwave Imaging of Large Inhomogeneous Bodies†

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Abstract

A frequency-hopping approach is proposed to process multifrequency CW microwave measurement data so that larger dielectric bodies for microwave imaging can be reconstructed with higher fidelity compared to a single-frequency reconstruction. The frequency hopping approach uses only data at a few frequencies, and hence can reduce data acquisition time in a practical system. Moreover, the frequency-hopping approach overcomes the effect of nonlinearity in the optimization procedure so that an algorithm is not being trapped in local minima. In this manner, larger objects with higher contrasts could be reconstructed without *a priori* information. We demonstrate the reconstruction of an object 10 wavelength in diameter with permittivity profile contrast larger than 1:2 without using *a priori* information.

1. Introduction

Microwave imaging involving large (c.f. wavelength) inhomogeneous bodies using inverse scattering technique is highly nonlinear. This is because the scattered fields are nonlinearly related to the inhomogeneity. The nonlinearity is a consequence of multiple scattering [1]. Therefore, as the body becomes large compared to the wavelength, or when the contrasts of the inhomogeneity become large, the nonlinear effect, or the multiple scattering

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effect becomes more pronounced.

However, this effect is less pronounced at lower frequencies [2,3]. Therefore, an inverse problem involving higher contrasts can be solved at lower frequencies. When the frequency becomes higher, the inverse problem becomes more nonlinear. An optimization approach is the robust way to solve the inverse scattering problem. However, due to the nonlinearity, the use of monofrequency data at a high frequency often results in the inverse algorithm being trapped in local minima due to the highly nonlinear nature of the problem. A way to rectify this problem is to obtain *a priori* knowledge about the scatterer, so that the highly nonlinear problem can be linearized about a different background to alleviate the nonlinear effect [4]. However, such *a priori* information is not available for many applications in general. Therefore, when no *a priori* information is assumed, it is difficult to invert data for inhomogeneous scatterer larger than two or three wavelengths when CW data are used.

On the other hand, we have noticed that when broadband data, or time-domain data are used, the inverted body could be as much as eight wavelengths in diameter [5,6]. The reason being that Born-type approximations are increasing good at lower frequencies. Therefore, the low frequency spectrum of the time-domain data is aiding in the linearization of the problem.

However, for many practical applications, high quality data are collected at CW by time averaging. Multifrequency data can thus be collected, but the measurement time is usually linearly proportional to the number of frequency points where the data are collected. In this paper, we show the successful use of a small set of CW data measured over discrete frequencies. By using the image reconstructed from low frequency data as the initial guess to the higher frequency problem, we find that the nonlinear effect can be mitigated. By slowly hopping from lower frequencies to higher frequencies, we can reconstruct objects which are as large as 10 wavelengths in diameter with high fidelity. The image reconstructed is much better than using the high-frequency data directly. Such a microwave imaging algorithm needs no *a priori* information about the inhomogeneous body.

2. Inverse scattering with CGFFT

In inverse scattering, one reconstructs the permittivity profile of the scatterer from a column vector of measurement data, $\boldsymbol{\Phi}^{sca}_{meas}$, collected at different combinations of transmitter and receiver locations [1-6]. The unknown scatterer is described by the object function $\delta\epsilon(\mathbf{r}') = \epsilon(\mathbf{r}') - \epsilon_b(\mathbf{r}')$ where $\epsilon(\mathbf{r}')$ is the unknown permittivity profile to be solved for, and $\epsilon_b(\mathbf{r}')$ is a guessed or estimated permittivity profile which is assumed known. The measured data are nonlinearly related to the object function because of multiple scattering [1]. A way to solve this nonlinear problem is to iteratively optimize a cost function, which is a measure of the difference of $\boldsymbol{\Phi}^{sca}_{meas}$, the measurement data, and $\boldsymbol{\Phi}^{sca}$, the simulation data from an estimated object profile $\epsilon_b(\mathbf{r}')$. The simulated data are generated by a rapid forward scattering solver. A cost function can be defined as

$$S(\boldsymbol{\epsilon}) = \frac{1}{2}\left(\left\|\boldsymbol{\Phi}^{sca}(\boldsymbol{\epsilon}) - \boldsymbol{\Phi}^{sca}_{meas}\right\|^2 + \delta_t \left\|\boldsymbol{\epsilon} - \boldsymbol{\epsilon}_b\right\|^2 \right), \tag{1}$$

where the column vectors $\boldsymbol{\epsilon}$ and $\boldsymbol{\epsilon}_b$ contain the discretized values of $\epsilon(\mathbf{r}')$ and $\epsilon_b(\mathbf{r}')$ at $\mathbf{r}'$ defined over a mesh. Note that the second norm in the above equation serves as the regularization term to circumvent the inherently ill-conditioned nature of the inverse scattering problems [1]. A Newton-type minimization method, the conjugate gradient method, is used to minimize the cost function. At each iteration, the problem is linearized, and the gradient of the functional is required to calculate the conjugate vector and the Hessian to find the step size. Hence, it is assumed that the functional changes quadratically with the object profile. In calculating the gradient and Hessian, the so called Fréchet derivative operator, $\overline{\mathbf{F}} = \dfrac{\partial \boldsymbol{\Phi}^{sca}}{\partial \boldsymbol{\epsilon}}$ has to be found [7]. The operator $\overline{\mathbf{F}}$ establishes a linear relationship between $\delta\boldsymbol{\Phi}$ and $\delta\boldsymbol{\epsilon}$, or

$$\delta\boldsymbol{\Phi} = \overline{\mathbf{F}} \cdot \delta\boldsymbol{\epsilon}. \tag{2}$$

In the above, $\delta\boldsymbol{\Phi} = \boldsymbol{\Phi}^{sca} - \boldsymbol{\Phi}^{sca}_{meas}$, and $\delta\boldsymbol{\epsilon} = \boldsymbol{\epsilon} - \boldsymbol{\epsilon}_b$. More explicitly,

$$\delta\Phi_j = \delta\phi_{lm} = \phi^{sca}(\mathbf{r}_{tl}, \mathbf{r}_{rm}, \boldsymbol{\epsilon}) - \phi^{sca}_{meas}(\mathbf{r}_{tl}, \mathbf{r}_{rm}), \tag{3}$$

$$F_{ji} = k_0^2 \Delta\mathbf{r}' g_b(\mathbf{r}_{rm}, \mathbf{r}_i) g_b(\mathbf{r}_i, \mathbf{r}_{tl}) \tag{4}$$

$$\delta\epsilon_i = \delta\epsilon(\mathbf{r}_i) = \epsilon(\mathbf{r}_i) - \epsilon_b(\mathbf{r}_i) \tag{5}$$

where j here stands for both subscripts l and m, and $g_b(\mathbf{r}, \mathbf{r}')$ is the Green's function of the inhomogeneous background, which is assumed to be the current object function profile with a permittivity of $\epsilon_b(\mathbf{r}')$. When (2) is written explicitly, it is

$$\begin{aligned}\delta\Phi_j = \delta\phi_{lm} &= \sum_i k_0^2 \Delta\mathbf{r}' g_b(\mathbf{r}_{rm}, \mathbf{r}_i) g_b(\mathbf{r}_i, \mathbf{r}_{tl}) \delta\epsilon(\mathbf{r}_i) \\ &\approx \int k_0^2 d\mathbf{r}' g_b(\mathbf{r}_{rm}, \mathbf{r}') g_b(\mathbf{r}', \mathbf{r}_{tl}) \delta\epsilon(\mathbf{r}_i) \end{aligned} \tag{6}$$

The above integral is also known as the distorted Born approximation of the volume integral equation of scattering, and the Fréchet derivative is hence directly obtained by such an approximation [3].

The unknown $\epsilon(\mathbf{r}')$ is found by minimizing (1). Then the value of $\epsilon(\mathbf{r}')$ is used as the new $\epsilon_b(\mathbf{r}')$. Consequently, $g_b(\mathbf{r}, \mathbf{r}')$ has to be updated in each iteration by solving a forward scattering problem. This method of solving the inverse problem, called the distorted Born iterative method, has a second order convergence rate [3,4], which is in contrast to the Born iterative method [2] with a first order convergence rate. An efficient forward solver, such as CG-FFT [8], is invoked to solve for $g_b(\mathbf{r}, \mathbf{r}')$. Since the solution of the previous step is used as an initial guess, it only needs a few inner iterations to converge in the forward scattering problems. Over all, this algorithm has an $O(N_o N_t N_i N \log_2 N)$ complexity*, where N_o is the number of outer iterations for solving (1), N_i is the number of inner iterations needed to solve the forward problem by a CG-type method, N_t is the number of transmitter locations used for the inverse problem, and N is the number of unknowns used in the forward scattering problem. For an analysis of the computational complexity, see [9].

3. Results and Conclusions

We have applied the CG-FFT algorithm to real experimental data [10,11]. The data, from an experimental setup in Barcelona, Spain, is based on a

* Here, O means "order of."

cylindrical array of 64 antennas equispaced on a circle 25 cm in diameter at 2.45 GHz. Better reconstruction could be obtained by using *a priori* information, but without it, this algorithm, using only monochromatic data, has apparently been trapped in a local minima.

Since the experimental data are not available for multifrequency, we demonstrate the frequency-hopping technique using synthetic data. The permittivity values for the multi-frequency synthetic data are derived from reference [12]. In this approach, low frequency data are first used to perform the image reconstruction, and the resultant image used as an initial guess for the next higher-frequency reconstruction. In Figure 1, we show the comparison of the image obtained by the frequency-hopping method, and that by using the high-frequency data directly. It is seen that the frequency-hopping method could accurately reconstruct the image quantitatively, while the monochromatic reconstruction is inaccurate. No *a priori* information is assumed for the image in both cases. The reconstruction for the imaginary part (not shown due to lack of space) is also superior in the frequency-hopping approach. The object is immersed in water as in [10].

In Figure 2, a ten-wavelength object is reconstructed using the frequency-hopping method. Previously, such a large object with such a contrast could not be reconstructed if only monochromatic data are available. The lowest frequency is chosen so that the object size is about 1.5 to 2 wavelengths, and the frequency can be roughly doubled until the highest frequency is reached.

In conclusion, we have developed a method of reconstructing the images of large objects in a quantitative manner by using multi-frequency information. The reconstruction of ten-wavelength object with contrasts larger than 1:2 was not possible previously, but with the frequency-hopping approach, this is now possible. Since the number of frequencies used is in general small, the data collection time could be reduced. In this reconstruction method, no *a priori* information is needed about the object.

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Figure Captions

Figure 1. The comparison of the reconstructions (image and profile of the real part of the permittivity) from (left) the frequency hopping approach where data at three frequencies (1, 2, and 3 GHz) are used, with (right) the monochromatic case where only the 3 GHz data is used. The reconstructed area is $5.56\lambda_w \times 5.56\lambda_w$, where λ_w is the wavelength in water. The reconstruction took 12 min on a CRAY-YMP. The pixel size is $0.16\lambda_w$.

Figure 2. The reconstruction of the real part of the permittivity for an object resembling a human arm model using the frequency-hopping approach where the frequencies used are 0.5, 0.9, 1.65 and 3 GHz. The reconstructed area is $10.2\lambda_w \times 10.2\lambda_w$. The reconstruction took 60 min on a CRAY-YMP. The pixel size is $0.16\lambda_w$.