FAST FAR FIELD APPROXIMATION FOR CALCULATING THE RCS OF LARGE OBJECTS †

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Abstract

A fast far field approximation (FAFFA) is developed to estimate RCS of conducting scatterers. This method accounts for the interaction between subscatterers in two ways depending on the electrical distance between the subscatterers. For subscatterers separated by large electrical distance, their interactions are computed in three stages: (1) Aggregation, which computes the total field at a group center due to the subscatterers of the group. (2) Translation, which translate the field from one group center to another. (3) Disaggregation, which distribute the field in a group center to each subscatterer in the group. Two strategies are employed to accelerate the computation in the above three stages. One is the use of far field approximation to simplify the computation in the translation stage; the other is the use of interpolation and smoothing techniques, which reduces the complexity of aggregation and disaggregation. The overall computational complexity for a matrix-vector multiplication is of the order of $N^{1.33}$ and the memory requirement is of order N. Numerical results show that this method can predict RCS that is very close to exact solution and that it can be applied to objects with very large electrical sizes.

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1. Introduction

In many practical high frequency scattering problems, the scatterers are very large compared to wavelength. When high frequency RCS is required, exact solution (by exact solution, we mean the solution such as the direct inversion of a MOM matrix or the iterative solution using the exact MOM matrix) is impossible with current computer resources. In this case, approximate methods are very helpful. Typical approximate techniques for high frequencies are geometrical optics (GO), physical optics (PO), geometrical theory of diffraction (GTD), and physical theory of diffraction (PTD). These approximations have been incorporated into the shooting and bouncing ray (SBR) method [1].

In this paper, we develop another RCS estimation technique called the fast far field approximation (FAFFA). First, the integral equation is discretized by the method of moments (MOM) [2], so that the scatterer is broken into N subscattering elements. The idea of FAFFA is to compute the interactions between elements using different methods depending on the electrical distance between the elements. When the distance is small, exact interaction is calculated; when the distance is large, a far field approximation is used. The field interactions are calculated iteratively using conjugate gradient (CG) method [3]. The iteration process is equivalent to taking into account the higher order terms in GO, PO or PTD approximations. After several iterations, we find the results to be greatly improved over that of a simple PO or GO. This algorithm has a computational complexity of $O(N^{1.33})$ for N unknown problems.

In the following, we first discuss the algorithm mathematically, and then give numerical results to validate the algorithm and show its applications to large size objects. The formulation is presented for electric field integral equation (EFIE) for TM polarization. But this algorithm can be applied also to combined field integral equation (CFIE) as well as to TE polarization. The numerical simulation consists of TE as well as TM cases. In both cases, CFIE is used to increase the convergence rate. In this paper, the time dependence of $e^{-i\omega t}$ is assumed and suppressed.

2. Formulation

Consider a TM (to z) plane wave incident on a two dimensional conducting cylinder in free space. The integral equation satisfied by the induced current J can be written as,

$$\frac{\omega\mu_0}{4}\int_S d\mathbf{r}' J(\mathbf{r}')H_0^{(1)}(k|\mathbf{r}-\mathbf{r}'|) = E^{inc}(\mathbf{r}), \tag{1}$$

where $H_0^{(1)}(x)$ is a 0-th order Hankel function of first kind. The integration is over the contour of the cylinder. $E^{inc}(\mathbf{r}) = E_0 \exp(-i\mathbf{k}^i \cdot \mathbf{r})$. The subscript z for J and E^{inc} is implied here. Moreover, $\mathbf{r} = \hat{x}x + \hat{y}y$ is a 2D vector, $\mathbf{k}^i = k(\hat{x}\cos\theta^i + \hat{y}\sin\theta^i)$, where θ^i is the incident angle, and $k = 2\pi/\lambda$ is the free space wave number.

To discretize Equation (1), we first divide the surface into strips, and assume that the induced current on each strip is a constant J_j. Thus the discretized form of Equation (1), according to the method of moments, is [2],

$$\sum_{j=1}^{N} g_{ij} J_j = E_i/(30\pi k), \qquad i = 1, 2, \cdots, N, \tag{2}$$

where,

$$g_{ij} = \begin{cases} \Delta_j H_0^{(1)}(kr_{ji}) & i \neq j, \\ \Delta_j \left[1 + i\frac{2}{\pi}\ln(0.163805k\Delta_i)\right], & i = j, \end{cases}$$

Δ_j is the strip width, $E_i = E^{inc}(\mathbf{r}_i)$ and $r_{ji} = |\mathbf{r}_i - \mathbf{r}_j|$.

Equation (2) can be solved using the conjugate gradient (CG) method [3]. The most costly part of a CG iteration is the performance of the matrix-vector multiply, i.e., to compute the summation on the left hand side of Equation (2) for $i = 1, 2, \cdots, N$. Hence, the operation count of a matrix-vector multiplication is of $O(N^2)$.

Various methods have been explored to accelerate the matrix-vector multiply [4–11]. In this paper, we describe an algorithm which approximately performs the matrix-vector multiply of Equation (2) with $O(N^{1.33})$ operations. The method makes use of the two-level algorithm as proposed in the original fast multipole method [4,7,8], but our two-level algorithm has lesser complexity compared to the original two-level fast multipole method. It is the natural approximation of the ray propagation fast multipole algorithm [9].

In the first step, we divide the N unknowns into groups as shown in Figure 1. The rules for the division are that: (1) each group should have roughly the same number of unknowns, and (2) the members of each group are as close to each other as possible. Assuming that each group has M unknowns, the number of groups will be N/M. In the following, we use index i and j to number of unknowns, and use G_l and $G_{l'}$ to denote groups l and l', respectively. With this notation, Equation (2) can be rewritten as,

$$\sum_{l'=1}^{N/M} \sum_{j \in G_{l'}} g_{ij} J_j = E_i, \qquad i \in G_l, l = 1, 2, \cdots, \frac{N}{M}. \tag{3}$$

Depending on the distance $r_{l'l}$, we define two regions for each group G_l. One is the near field (NN) region, the other is the far field (FF) region. A group $G_{l'}$ is said to be in the far field region of group G_l if the distance between the two groups is larger than a certain quantity. Otherwise, $G_{l'}$ is in the near field region of G_l. Consequently, the summation in Equation (3) can be split into two parts: one corresponding to the near field (NN) contribution, while the other corresponding to the far field (FF) contribution, namely,

$$\begin{aligned} \sum_{l'\in NN}\sum_{j\in G_{l'}} g_{ij}J_j + \sum_{l'\in FF}\sum_{j\in G_{l'}} J_j H_0^{(1)}(kr_{ji})\Delta_j = E_i/(30\pi k), \\ i =\in G_l, l = 1, 2, \cdots, \frac{N}{M}. \end{aligned} \tag{4}$$

The far field contribution, which is proportional to,

$$I_{li} = \sum_{l'\in FF}\sum_{j\in G_{l'}} J_j H_0^{(1)}(kr_{ji}), \tag{5}$$

can be calculated approximately by first applying the large argument approximation to $H_0^{(1)}(kr_{ji})$:

$$H_0^{(1)}(kr_{ji}) \sim \sqrt{\frac{2}{\pi i}}\frac{e^{ikr_{ji}}}{\sqrt{kr_{ji}}}, \qquad kr_{ji} \gg 1. \tag{6}$$

Next, we write

$$\mathbf{r}_{ji} = \mathbf{r}_{jl'} + \mathbf{r}_{l'l} + \mathbf{r}_{li}.$$

Thus, if $r_{l'l} \gg r_{jl'}$ and $r_{l'l} \gg r_{li}$, then,

$$r_{ji} \sim r_{l'l} + \hat{r}_{l'l}\cdot\mathbf{r}_{li} + \hat{r}_{l'l}\cdot\mathbf{r}_{jl'}. \tag{7}$$

Substituting Equations (6) and (7) into Equation (5), we have

$$I_{li} = \sum_{l'\in FF}\sqrt{\frac{2}{\pi i}}\frac{e^{ikr_{l'l}}}{\sqrt{kr_{l'l}}}e^{ik\hat{r}_{l'l}\cdot\mathbf{r}_{li}}\sum_{j\in G_{l'}} J_j e^{ik\hat{r}_{l'l}\cdot\mathbf{r}_{jl'}} = \sum_{l'\in FF} e^{ik\hat{r}_{l'l}\cdot\mathbf{r}_{li}}\sqrt{\frac{2}{\pi i}}\frac{e^{ikr_{l'l}}}{\sqrt{kr_{l'l}}}b_{l'l}, \tag{8}$$

where,

$$b_{l'l} = \sum_{j\in G_{l'}} J_j e^{ik\hat{r}_{l'l}\cdot\mathbf{r}_{jl'}}. \tag{9}$$

Calculation of Equation (8) for all pairs of (l, i) requires $NM + N^2/M$ operations. The near field operation count is proportional to NM, and the total CPU is

$$CPU \sim C_1 NM + C_2 M \left(\frac{N}{M}\right)^2. \tag{10}$$

where, C_1 and C_2 are constants. Optimization yields $M = \sqrt{N}$ and $CPU \sim N^{1.5}$.

Equation (8) consists of three steps. The first step is called aggregation. It is represented by the summation in Equation (9). The second step is translation, corresponding to,

$$b'_{l'l} = \sqrt{\frac{2}{\pi i}} \frac{e^{ikr_{l'l}}}{\sqrt{kr_{l'l}}} b_{l'l} \tag{11}$$

Finally, the third step is disaggregation, corresponding to,

$$I_{li} = \sum_{l' \in FF} b'_{l'l} e^{ik\hat{r}_{l'l'} \cdot \mathbf{r}_{li}}. \tag{12}$$

The reduction in operation from N^2 to $N^{1.5}$ is attributed to replacing the translation matrix by a scalar number.

To further reduce the operation count, we consider the aggregation step more carefully. Computation of Equation (9) for all l, l' requires N^2/M operations. However, we know that $b_{l'l}$ depends only on $\hat{r}_{l'l}$ which is a function of an angle $\alpha \in (-\pi, \pi)$. The oscillation of $b_{l'l}$ with respect to α will depend on $\max(kr_{jl'})$ which is the electrical size of a group. Hence, it inspires us to compute the summation in Equation (8) in two steps. The first step is to compute it on equal samples of α over $[-\pi, \pi]$, and then to compute $b_{l'l}$ by interpolating with the known values on the samples. The number of α samples can be chosen to be proportional to the number of unknowns in the group.

In order to reduce the total computational cost, the cost for the disaggregation stage (Equation (9)) should also be reduced. This can be achieved by smoothing [12], which is just the converse of interpolation. In this manner, the operation count or the $CPU \sim N^{4/3}$.

The above method can easily be extended to the TE polarization, and also to the combined field integral equation.

3. Numerical Simulations

In this Section, we give numerical examples to compare the calculated RCS using fast far field approximation (FAFFA) with the result using CG

via the exact moment matrix. All computations are conducted on SPARC-10 workstation and in all the examples, combined field integral equation (CFIE) is used. The numerical results include both TM and TE cases.

Our first example is a 2D corner reflector with arm lengths of $A = 30\lambda$, $B = 72\lambda$ and corner angle of $C = 120^0$. The E_z polarized plane wave is incident at $\theta^i = 45^0$, as shown in Figure 2. The result from fast far field approximation agrees well with the exact solution (The exact solution is obtained using the same iteration algorithm as FAFFA but with the exact moment matrix). For comparison, the RCS predicted using PO is also ploted on the figure. It is seen that the peak at 100^0 is missed by PO because pure PO does not account for multiple reflections. On the contrary, PO predicts a peak at 145^0 which is the reflection direction of arm A. However, this reflection is actually blocked by arm B so that the real RCS in the reflection direction of arm A is much smaller than that predicted by PO.

Figure 3 shows the RCS comparison for TE polarized case for an ogive cylinder whose parameters are $A = B = 1\lambda$, $C = 8\lambda$. Both figures show that the agreements are very good between the results of FAFFA and the exact solution.

Finally, we give an example to show the ability of the fast far field approximation to deal with objects with very large electrical size. Figure 4 plots the RCS of the object with triangular cross section. The equilateral triangle are of sides 600, 2,400 and 2,400 wavelengths. The incident wave at 0^o is from the sharp apex of the triangle. In this large problem, $N = 54,000$ while the number of group is $1,428$. A CG method with 25 iterations consuming 5 hrs of SPARC 10 CPU time is needed to produce this Figure.

With the above comparisons, We verified that the fast far field approximation automatically takes into account the edge diffraction, surface waves and multi-reflections, etc. This feature, plus its very low complexity, yield a very efficient approach for accurate RCS prediction of large and complex targets.

4. Conclusions

A fast far field approximation (FAFFA) for RCS calculation of larger size targets is developed. This method considers near field and far field interactions differently so that the complexity for performing a matrix-vector multiplication is reduced from $O(N^2)$ to $O(N^{1.33})$ and the memory requirement is of $O(N)$ for N unknown problems. The method differs from the exact method since far field approximation is used. It also differs from a simple PO since the solution is calculated iteratively so that the higher order scattering field, such as edge diffraction, surface wave and multiple-reflection,

are automatically taken into account. Numerical simulations show that this method predicts RCS which are very close to the exact solution, and is greatly improved over that from applying pure PO. Also, due to the reduced computational complexity and memory requirement, the method can be applied to calculate RCS for targets with very large electrical sizes.

References

[1] Ling, H., R. C. Chou and S. W. Lee, "Shooting and bouncing rays: calculating the RCS of an arbitrarily shaped conductor," *IEEE Trans. Antennas and Propagat.,* vol. 37, pp. 194-205, 1989.

[2] Harrington, R. F., *Field computation by moment method*, Krieger, Marlabar, Florida, 1982.

[3] Press, W. H., S. A. Teukolsky, W. T. Vetterling and B. P. Flannery, *Numerical Recipes*, Cambridge University Press, 1992.

[4] Rokhlin, V., "Rapid solution of integral equations of scattering theory in two dimensions," *J. Comput. Phys.*, 86, (2), pp. 414-439, 1990.

[5] Canning, F. X., "Transformations that produce a sparse moment method matrix," *J. Electromagnetic Waves and Applications,* vol. 4, pp. 903-913, 1990.

[6] Anderson, C. R., " An implementation of the fast multipole method without multipoles," *SIAM J. Sci. Stat. Comput.,* vol. 13, no. 4, pp. 923-947, 1992.

[7] Coifman, R., V. Rokhlin, and S. Wandzura, "The fast multipole method for the wave equation: a pedestrian prescription," *IEEE Antennas Propagat. Mag.,* vol. 35, no. 3, pp.7-12, June 1993.

[8] Lu, C. C. and W. C. Chew, "Fast algorithm for solving hybrid integral equations," *IEEE Proc., Part-H,* vol. 140, No. 6, pp. 455-460, Dec. 1993.

[9] Wagner, R. L. and W. C. Chew, "A ray propagation fast multipole algorithm," *Micro. Opt. Tech. Lett.,* vol. 7, No. 10, pp. 435-438, July 1994.

[10] Lu, C. C. and W. C. Chew, "A multilevel algorithm for solving a boundary integral equations of wave scattering," *Micro. Opt. Tech. Lett.,* vol. 7, No. 10, pp. 466-470, July 1994.

[11] Michielssen, E. and A. Boag, "Multilevel evaluation of electromagnetic fields for the rapid solution of scattering problems," *Micro. Opt. Tech. Lett.*, in press.

[12] Brandt, A., "Multilevel computations of integral transforms and particle interactions with oscillatory kernels," *Comput. Phys. Commun.,* vol. 65, pp. 24-38, 1991.

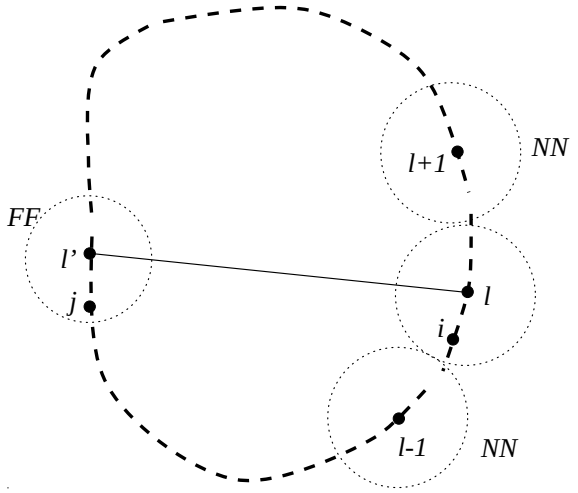

Figure 1 The discretized scatterer is divided into groups. The interaction between groups separated by large distances is calculated using far field approximation while the near field groups' interaction is calculated exactly. Group l, groups $l-1$, l, and $l+1$ are considered as near field groups (NN); while group l', it is considered as far field group (FF).

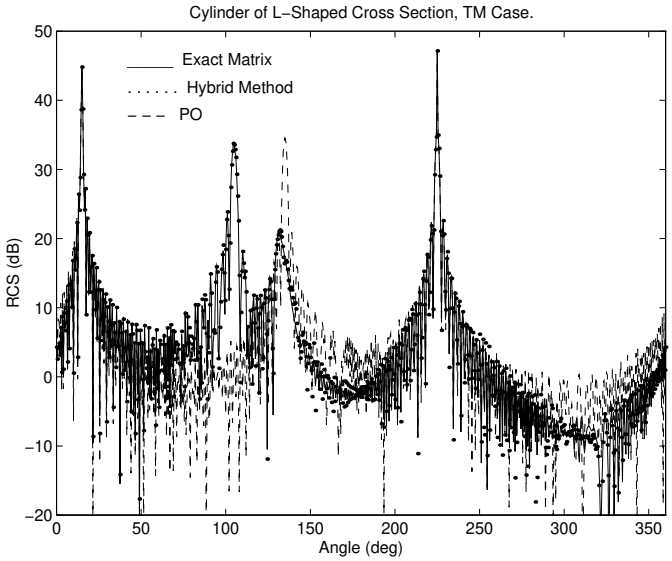

Figure 2 The RCS of a 2D corner reflector with arms $A = 30\lambda$, $B = 72\lambda$ and corner angle $C = 120^0$. The plane wave (TM) incident angle is $\theta^i = 45^0$. The iteration number here is 10.

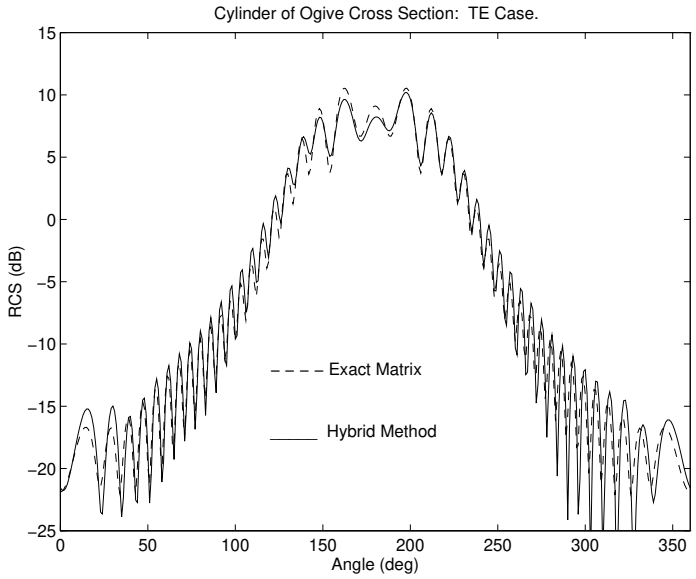

Figure 3 The RCS of a cylinder of ogive cross section with hight $A = B = 1\lambda$ and length $C = 10\lambda$. The plane wave (TE) incident angle is $\theta^i = 0$. The iteration number = 10.

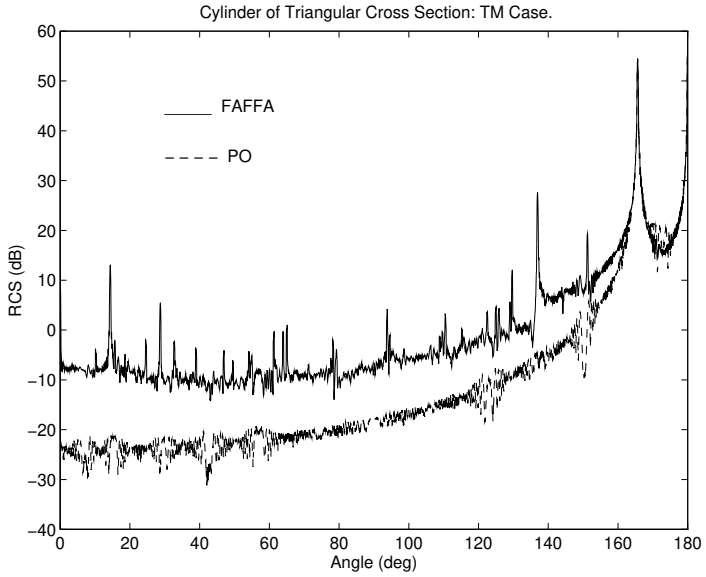

Figure 4 The RCS of a large triangular cylinder with sides of 600, 2,400 and 2,400 λ. The plane wave (TM) incident angle is $\theta^i = 0$ from the sharp apex. The iteration number = 25.