FAST FAR FIELD APPROXIMATION FOR CALCULATING THE RCS OF LARGE OBJECTS †

Cai-Cheng Lu and Weng Cho Chew
Department of Electrical and Computer Engineering
University of Illinois
Urbana, IL 61801

1. Introduction

In many practical high frequency scattering problems, the scatterers are very large compared to wavelength. When high frequency RCS is required, exact solution is impossible with current computer resources. In this case, approximate methods are very helpful. Typical approximate techniques are geometric optics (GO), physical optics (PO), geometric theory of diffraction (GTD), and physical theory of diffraction (PTD). These approximations have been incorporated into the shooting and bouncing ray (SBR) method [1].

In this paper, we develop another RCS estimation technique called the fast far field approximation (FAFFA). The idea of this method is to compute the interactions between elements using different methods depending on the electrical distance between the elements. When the distance is small, exact interaction is calculated; when the distance is large, a far field approximation is used. The field interactions are calculated iteratively using conjugate gradient (CG) method. The iteration process is equivalent to taking into account the higher order terms in GO, PO or PTD approximations. After several iterations, we find the results to be greatly improved over that of a simple PO or GO. On the other hand, this method can deal with small scale target geometry variations since it computes the near field interaction exactly hence it is more flexible to be applied to arbitrary complex targets. This algorithm has a computational complexity of $O(N^{1.33})$ and memory requirement of $O(N)$ for N unknown problems.

In the following, we first discuss the algorithm mathematically, and then give numerical results to validate the algorithm and show its applications to large size objects. The formulation is presented for electric field integral equation (EFIE) for TM polarization. The numerical simulation contains both 2D and 3D results. In this paper, the time factor of $e^{-i\omega t}$ is assumed

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2. Formulation

Consider a TM (to z) plane wave incident on a two dimensional conducting cylinder in free space. The integral equation satisfied by the induced current J can be written as,

$$\frac{\omega\mu_0}{4}\int_S d\mathbf{r}' J(\mathbf{r}') H_0^{(1)}(k|\mathbf{r}-\mathbf{r}'|) = E^{inc}(\mathbf{r}), \tag{1}$$

where $H_0^{(1)}(x)$ is a 0-th order Hankel function of first kind. The integration is over the contour of the cylinder. $E^{inc}(\mathbf{r}) = E_0 \exp(-i\mathbf{k}^i \cdot \mathbf{r})$. The subscript z for J and E^{inc} is implied here. Moreover, $\mathbf{r} = \hat{x}x + \hat{y}y$ is a 2D vector, $\mathbf{k}^i = k(\hat{x}\cos\theta^i + \hat{y}\sin\theta^i)$, where θ^i is the incident angle, and $k = 2\pi/\lambda$ is the free space wave number.

To discretize Equation (1), we first divide the surface into strips, and assume that the induced current on each strip is a constant J_j. Thus the discretized form of Equation (1), according to the method of moments, is [2],

$$\sum_{j=1}^{N} g_{ij} J_j = E_i/(30\pi k), \qquad i = 1, 2, \cdots, N, \tag{2}$$

where,

$$g_{ij} = \begin{cases} \Delta_j H_0^{(1)}(kr_{ji}) & i \neq j, \\ \Delta_j \left[1 + i\frac{2}{\pi}\ln(0.163805k\Delta_i)\right], & i = j, \end{cases}$$

Δ_j is the strip width, $E_i = E^{inc}(\mathbf{r}_i)$ and $r_{ji} = |\mathbf{r}_i - \mathbf{r}_j|$.

Equation (2) can be solved using the conjugate gradient (CG) method [3]. The most costly part of a CG iteration is the performance of the matrix-vector multiply, i.e., to compute the summation on the left hand side of Equation (2) for $i = 1, 2, \cdots, N$. Hence, the operation count of a matrix-vector multiplication is of $O(N^2)$.

Various methods have been explored to accelerate the matrix-vector multiply [4–10]. In this paper, we describe an algorithm which approximately performs the matrix-vector multiply of Equation (2) with $O(N^{1.33})$ operations. The method makes use of the two-level algorithm as proposed in the original fast multiple method [4,7,8], but we arrive at a two level algorithm with lesser complexity compared to the original two-level fast multiple method.

In the first step, we divide the N unknowns into groups as shown in Figure 1. The rules for the division are that: (1) each group should have roughly the same number of unknowns, and (2) the members of each group are as close to each other as possible. Assuming that each group has M unknowns, the number of groups will be N/M. In the following, we use index i and j to number unknowns, and use G_l and $G_{l'}$ to denote groups l and l', respectively.

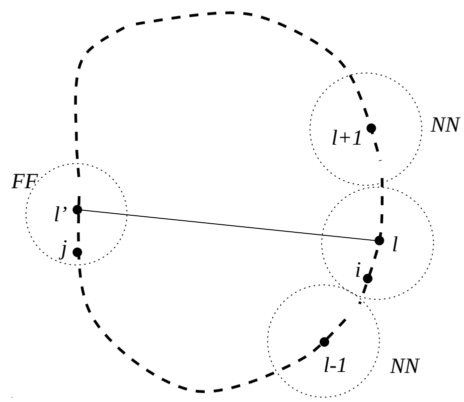

Figure 1 The discredited scatterer is divided into groups. The interaction between groups with large distance is calculated using far field approximation; the near field groups' interaction is calculated exactly. For group l, groups $l-1$, l, and $l+1$ are considered as near field groups (NN); For group l', it is considered as far field group (FF).

With this notation, Equation (2) can be rewritten as,

$$\sum_{l'=1}^{N/M} \sum_{j \in G_{l'}} g_{ij} J_j = E_i, \qquad i \in G_l, l = 1, 2, \cdots, \frac{N}{M}. \tag{3}$$

Depending on the distance $r_{l'l}$, we define two neighboring regions for each group G_l. One is the near field (NN) region, the other is the far field (FF) region. A group $G_{l'}$ is said to be in the far field region of group G_l if the distance between the two groups is larger than a certain quantity. Otherwise, $G_{l'}$ is in the near field region of G_l. Consequently, the summation in Equation (3) can be split into two parts: one corresponding to the near field (NN) contribution, while the other corresponding to the far field (FF) contribution, namely,

$$\begin{gathered} \sum_{l' \in NN} \sum_{j \in G_{l'}} J_j H_0^{(1)}(kr_{ji}) \Delta_j + \sum_{l' \in FF} \sum_{j \in G_{l'}} g_{ij} J_j = E_i/(30\pi k), \\ i = \in G_l, l = 1, 2, \cdots, \frac{N}{M}. \end{gathered} \tag{4}$$

The far field contribution, which is proportional to,

$$I_{li} = \sum_{l' \in FF} \sum_{j \in G_{l'}} J_j H_0^{(1)}(kr_{ji}), \tag{5}$$

can be calculated approximately by first applying the large argument approximation to $H_0^{(1)}(kr_{ji})$:

$$H_0^{(1)}(kr_{ji}) \sim \sqrt{\frac{2}{\pi i}} \frac{e^{ikr_{ji}}}{\sqrt{kr_{ji}}}, \qquad kr_{ji} \gg 1. \tag{6}$$

Next, we write

$$\mathbf{r}_{ji} = \mathbf{r}_{jl'} + \mathbf{r}_{l'l} + \mathbf{r}_{li}.$$

Thus, if $r_{l'l} \gg r_{jl'}$ and $r_{l'l} \gg r_{li}$, then,

$$r_{ji} \sim r_{l'l} + \hat{r}_{l'l} \cdot \mathbf{r}_{li} + \hat{r}_{l'l} \cdot \mathbf{r}_{jl'}. \tag{7}$$

Substituting Equations (6) and (7) into Equation (5), we have

$$I_{li} = \sum_{l' \in FF} \sqrt{\frac{2}{\pi i}} \frac{e^{ikr_{l'l}}}{\sqrt{kr_{l'l}}} e^{ik\hat{r}_{l'l} \cdot \mathbf{r}_{li}} \sum_{j \in G_{l'}} J_j e^{ik\hat{r}_{l'l} \cdot \mathbf{r}_{jl'}} = \sum_{l' \in FF} e^{ik\hat{r}_{l'l} \cdot \mathbf{r}_{li}} \sqrt{\frac{2}{\pi i}} \frac{e^{ikr_{l'l}}}{\sqrt{kr_{l'l}}} b_{l'l}, \tag{8}$$

where,

$$b_{l'l} = \sum_{j \in G_{l'}} J_j e^{ik\hat{r}_{l'l} \cdot \mathbf{r}_{jl'}}. \tag{9}$$

Calculation of Equation (8) for all pairs of (l, i) requires $NM + N^2/M$ operations. The near field operation count is proportional to NM, and the total CPU is

$$CPU \sim C_1 NM + C_2 M \left(\frac{N}{M} \right)^2. \tag{10}$$

where, C_1 and C_2 are constants. Optimization yields $M = \sqrt{N}$ and $CPU \sim N^{1.5}$.

Equation (8) consists of three steps. The first step is called aggregation. It is represented by the summation in Equation (9). The second step is translation, corresponding to,

$$b'_{l'l} = \sqrt{\frac{2}{\pi i}} \frac{e^{ikr_{l'l}}}{\sqrt{kr_{l'l}}} b_{l'l} \tag{11}$$

Finally, the third step is disaggregation, corresponding to,

$$I_{li} = \sum_{l' \in FF} b'_{l'l} e^{ik\hat{r}_{l'l} \cdot \mathbf{r}_{li}}. \tag{12}$$

The reduction in operation from N^2 to $N^{1.5}$ is attributed to replacing the translation matrix by a scalar number.

To further reduce the operation count, we consider the aggregation step more carefully. Computation of Equation (9) for all l, l' requires N^2/M operations. However, we know that $b_{l'l}$ depends only on $\hat{r}_{l'l}$ which is a function of an angle $\alpha \in (-\pi, \pi)$. The oscillation of $b_{l'l}$ with respect to α will depend on $\max(kr_{jl'})$ which is the electrical size of a group. Hence, it motivated us to compute the summation in Equation (8) in two steps. The first step is to compute it on equal samples of α over $[-\pi, \pi]$, and then to compute $b_{l'l}$ by interpolating with the known values on the samples. The number of α samples can be chosen to be proportional to the number of unknowns in

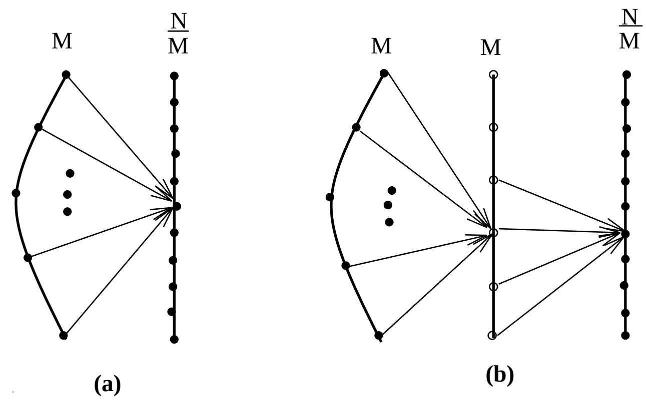

Figure 2 (a). Direct aggregation. (b). Aggregation using interpolation.

the group. The interpolation process is shown in Figure 2. Figure 2a shows the original operation which is the direct aggregation. Figure 2b shows the aggregation using interpolation. It is clear that if N/M is larger than M, then interpolation procedure is more efficient.

In order to reduce the total computational cost, the cost for the disaggregation stage (Equation (9)) should also be reduced. This can be achieved by smoothing [11], which is just the converse of interpolation. In this manner, the operation count or the $CPU \sim N^{4/3}$.

The above method can easily be extended to the TE polarization, and also to the combined field integral equation.

3. Numerical simulations

In this Section, we give numerical examples to compare the calculated RCS using fast far field approximation (FAFFA) with the result by CG using exact moment matrix. In all the examples, combined field integral equation (CFIE) is used. The numerical results include both TM and TE cases.

Our first example is a 2D open ended cavity. The cavity is of length, width and thickness 10m, 5m and 0.5m, respectively. The E_z polarized plane wave is incident at $\theta^i = 30^0$, as shown in Figure 3. The result from fast far field approximation agrees well with the exact solution.

The second example, shown in Figure 4, is the RCS of a 3D 90^o dihedral which consists of two flat plate of same size (1m $\times$ 5m, 1m side as the common edge). Vertical polarized plane wave is incident from the direction of $(\theta^i, \phi^i) = (45^o, 270^o)$ with a frequency 300 MHz.

The above two figures show that FAFFA takes into account of multiple reflections such as in corner reflector and in cavity like targets. Our next example shows that FAFFA will also take into account the small scale target geometry variations. Figure 5 plots the RCS of a two dimensional cylinder whose cross section is also shown in the figure. The geometry variation is in less than a wavelength scale.

¿From Figures 3 to 5, one can see that the RCS predicted by FAFFA are very close to the exact solution. The advantage of FAFFA is that it has lower computational complexity ($O(N^{4/3})$) and small memory requirement ($O(N)$). Hence it can be applied to predict RCS for targets with very large electrical size. Figures 6 and 7 are two examples.

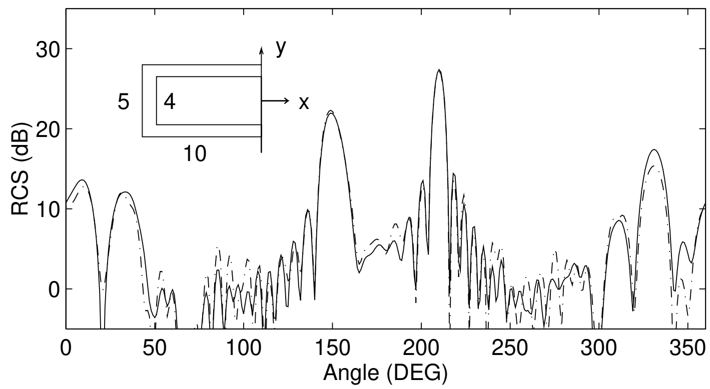

Figure 3 RCS of a 2D open ended cavity.

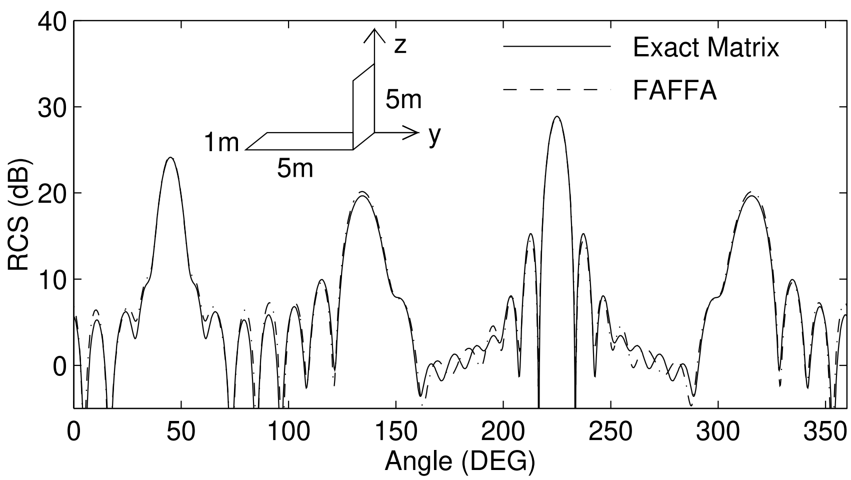

Figure 4 The RCS of a 3D dihedral corner reflector.

4. Conclusions

A fast far field approximation (FAFFA) for RCS calculation of larger size targets is developed. This method considers near field and far field interactions differently so that the complexity for performing a matrix-vector multiplication is reduced from $O(N^2)$ to $O(N^{1.33})$ and the memory requirement is of $O(N)$ for N unknown problems. The method differs from exact method since far field approximation is used. It also differs from pure PO since the solution is calculated iteratively so that the higher order scattering field, such as edge diffraction, surface wave and multiple-reflection, are automatically taken into account. Numerical simulations show that this method predicts

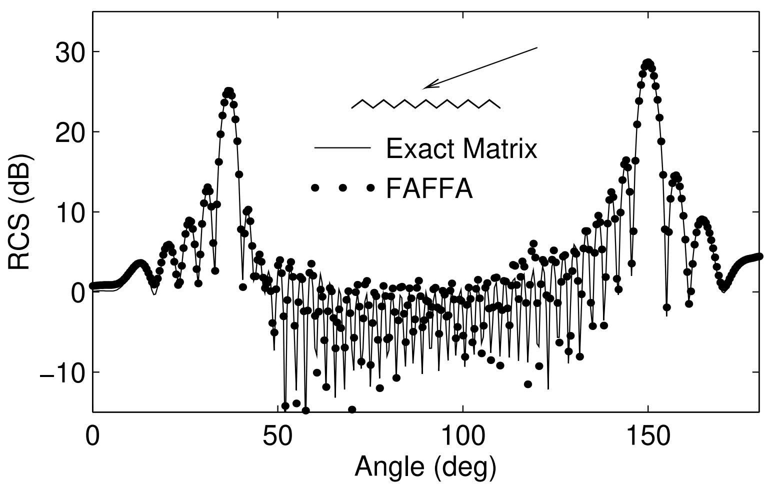

Figure 5 The RCS of a 2D cylinder with zigzag contour of cross section.

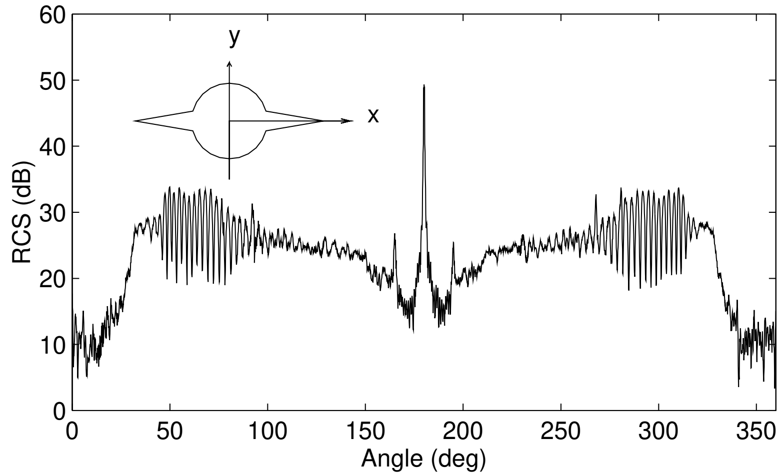

Figure 6 The RCS of a 2D complex target. Plane wave of frequency 300 MHz is incident from $\theta^i = 0^o$ and is polarized in z direction. $N = 52,788$.

RCS which are very close to the exact solution, and is greatly improved over that from applying pure PO. Also, due to the reduced computational complexity and memory requirement, the method can be applied to calculate RCS for targets with very large electrical size.

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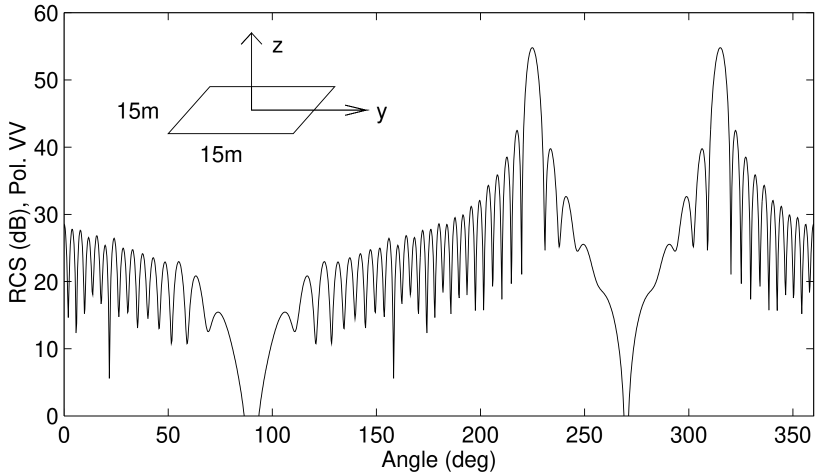

Figure 7 The RCS of a 3D square plate of size 15m×15m. Plane wave incident at $\theta^i = 45^o$, $\phi^i = 0^o$ and is of frequency 300 MHz. $N = 44,700$.

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