FAST ILLINOIS SOLVER CODE (FISC) †

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ABSTRACT—- FISC (Fast Illinois Solver Code) is designed to compute RCS of a target described by a triangular facet file. The problem is formulated by the method of moments (MoM), where the RWG (Rao, Wilton, and Glisson) basis functions are used. The resultant matrix equation is solved iteratively by the conjugate gradient (CG) method. The multilevel fast multipole algorithm (MLFMA) is used to speed up the matrix-vector multiply in CG. Both complexities for the CPU time per iteration and memory requirements are of $O(N\log N)$, where N is the number of unknowns.

1. Introduction

The fast multipole method is an efficient way to perform matrix-vector multiplies whereby the field at each particle due to every other particle is calculated for all particles in an ensemble of N particles. Ordinarily, this would require $O(N^2)$ calculations. With the fast multipole method, this can be reduced to $O(N)$ or $O(N\log N)$ depending on the spatial distribution density of the particles and implementation. The fast multipole method for static problems has been of interest to astrophysicists for calculating Coulombic interaction between stars in galaxies. One of the first such algorithms is due to Barnes and Hut [1]. Later, Rokhlin and Greengard [2,3] described a more efficient method of performing fast multipole calculation for static problems.

Due to the oscillatory nature of dynamic fields, the original fast multipole algorithm for statics does not expedite matrix-vector multiplies for dynamic fields. This is because the number of multipoles needed to represent a field accurately depends on the size of the source distribution compared to the wavelength. The larger the size of the source distribution, the more the number of multipoles needed to approximate the field well, irrespective of the distance of the observation point from the sources.

Rokhlin [4] first suggested a diagonal form of the translation matrices in order to achieve reduction in computational complexity for surface scatterers. This was first demonstrated for acoustic problems. Then Lu and Chew [5] demonstrated the fast multipole concept for vector electromagnetic problems in two dimensions. Coifman, Rokhlin and Wandzura [6] furnished a lucid description of the fast multipole method for three dimensional electromagnetic problems. An implementation of the algorithm was later provided by Song and Chew [7] for 3D vector electromagnetic scattering problems.

Unlike static problems, the fast multipole method for dynamic problems cannot be easily

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extended to the multilevel due to the oscillatory nature of the field. A method of treating the oscillatory field was suggested by Brandt [8] using interpolation and anterpolation. The idea of interpolation and anterpolation was later combined with the fast multipole method to arrive at a multilevel fast multipole algorithm (MLFMA) by Lu and Chew [9]. This was later generalized to three-dimensional vector electromagnetic problems by Song and Chew [10]. This results in an $O(N \log N)$ algorithm for performing matrix-vector multiplies for surface scatterers, and $O(N)$ algorithm for volumetric scatterers. Meanwhile, an $O(N \log^2 N)$ method of interpolation and smoothing for a multilevel fast multipole method using group-theoretic technique has been proposed by Dembart and Yip [11]. Another form of the multilevel algorithm has also been proposed by Michielssen and Boag albeit with $O(N \log^2 N)$ complexity [12].

Because the multilevel fast multipole algorithm (MLFMA) expedites matrix-vector multiplies, it can be used to speed up iterative solutions of scattering problems. We have combined MLFMA with the conjugate gradient (CG) and the biconjugate gradient (BCG) methods to arrive at efficient solvers for matrix equations arising from integral equation of scattering. The computational and memory-requirement complexities of this algorithm are both $O(N \log N)$ for surface scatterers. The total CPU time of the matrix solver is hence proportional to $N_{\text{iter}} N \log N$. Compared to traditional matrix solvers requiring $O(N^2)$ memory, and $N_{\text{iter}} N^2$ CPU time for iterative solvers, and $O(N^3)$ CPU time for LUD, this is a vast improvement, especially for large problems. Therefore, large problems that previously require the resources of a supercomputer to solve, can now be solved on a workstation-size computer.

FISC (Fast Illinois Solver Code) is designed to be an industrial-strength code using the most current technology from the method of moments (MoM) [13,14,15] and MLFMA. The method of moments is used carefully to develop a matrix equation from the integral equation. Both curvilinear quad-patch and tri-patch are used. Presently, the tri-patch is restricted to flat facets, but this restriction will be lifted in the future. Careful integrations are performed to account for singularities and near singularities in the evaluation for the matrix elements.

The code was first designed to model the scattering solution from a complex metallic target like an aircraft, so that its RCS (radar cross section) can be ascertained. Presently, impedance boundary conditions, thin dielectric sheets, and resistive boundary conditions can be modeled by the code. In the future, we hope to adapt the code to model material-coated targets as well as radiation of antennas from complex platforms like an aircraft and automobile.

2. Complexity and Accuracy

To test the complexity, we calculate the electromagnetic scattering from a conducting sphere solving the combined field integral equation (CFIE). The machine used is SGI Power Challenge with 4 processors (R8000) and 2 GB of memory. But only one processor is used for the simulations in this paper. The radius of the sphere is from 0.75 λ to 12 λ, number of unknowns (N) is from 2,352 to 602,112, and number of levels in MLFMA is from 3 to 7. The CPU time per iteration and memory requirement are plotted in Figure 1 as functions of the number of unknowns. Two curves, $8 \times 10^{-5} N \log N$ and $2.7 \times 10^{-3} N$, are also plotted on the same figure for comparison.

In Figure 2, we plot the $\theta\theta$ bistatic RCS of a conducting sphere of radius 1 m at 3.6 GHz (the diameter is 24 λ). A total of 602,112 unknowns with a seven-level MLFMA is used. It takes 12 hours on the Power Challenge using one processor [6 hours for set up (filling matrix and calculating plane wave expansions), 4 hours for 41 iterations to reach 0.001 normalized residual error, and 2 hours for calculating 1,201 points of bistatic RCS]. A good agreement between our result and the Mie series solution is observed. The RMS error is 0.3 dB for all 1,201 points, and 0.08 dB for elevation angles from 90 to -30 degrees (θ is from 0 to 120 degrees, the incident angle is 0 degree).

Figure 3 shows the monostatic RCS for the aircraft (VFY 218) at 100 MHz as functions of azimuth angle in the horizontal plane using the Rockwell flat triangular patch model for HH polarization. The wings of the VFY 218 are on the x-y plane (horizontal plane). Zero degree corresponds to incidence angle on the nose. A five-level MLFMA is used. The measurement data are from H. T. G. Wang, M. L. Sanders, and A. Woo in Naval Air Warfare Center [16]. Good agreement between the numerical results and the measurements is observed. For this 9,747 unknown problem, FISC needs 102 MB of memory for this single-precision code and requires 3 hours of the CPU time on a DEC Alpha workstation for 181 incident angles. In contrast, the LUD solution is estimated to need 800 MB of memory and 10 hours of the CPU time for LUD, and $O(N^2)$ calculations for each incident angle. We estimate that only for 1,000 incident angles, FISC would need the same CPU time as the LUD solution. But it needs memory (102 MB) much less than the LUD solution (800 MB). The comparison is more in favor of FISC when N becomes larger.

3. Capabilities

3.1. Integral Equations

Three kinds of integral equations [17] are used in FISC: electric field integral equation (EFIE), magnetic field integral equation (MFIE), and combined field integral equation (CFIE). We implement Galerkin's method and line matching method in FISC. In Galerkin's method, the testing functions are the same as the basis functions, the RWG basis [13]. In line matching, the testing functions are constant along the line joining the centers of two adjacent patches.

3.2. Boundary Conditions

In addition to PEC, FISC can deal with impedance boundary conditions (IBC), resistive sheets (R-card) and thin dielectric sheet (TDS).

When the coating material has a large refractive index ($|\sqrt{\epsilon\mu/(\epsilon_0\mu_0)}|$, where ϵ and μ are the permittivity and permeability of the coating material), the wave inside the coating sheet propagates approximately in the direction normal to the sheet. Then, the incident angle dependence of the impedance can be neglected. If the material is lossy or the coating sheet is very thin, it can be modeled as IBC with

$$\eta_s = j\sqrt{\frac{\mu}{\epsilon}}\tan\left(\sqrt{\frac{\epsilon\mu}{\epsilon_0\mu_0}}k_0 d\right),$$

where k_0 is the wavenumber in the free space, and d is the thickness. IBC also can be used for lossy, electrically large scatterers, such as the earth and the ocean surface. The scatterer is modeled with IBC using

$$\eta_s = \sqrt{\frac{\mu}{\epsilon}}.$$

In Figure 4, we plot the bistatic RCS for a sphere with 1 m radius at 300 MHz as functions of elevation angle $(90 - \theta)$. The impedance of impedance boundary condition is $100 + j37.7$

ohms. The sphere is described by 1,568 flat triangular facets, and three-level MLFMA is used. A good agreement between Mie series and numerical results by FISC is observed.

A thin dielectric sheet can be approximated by an impedance sheet

$$Z = \frac{1}{j\omega d \Delta\epsilon},$$

where d is its thickness, and $\Delta\epsilon = \epsilon - \epsilon_b$, where ϵ_b is the permittivity of the background.

3.3. Initial Guess

For iterative solutions of monostatic RCS, different incident angles require different iterative solutions. Since a small change in the incident angle corresponds to a small change in the current, we use the current solution from the previous angle with phase correction as the initial guess for the next angle. This technique reduces the number of iterations significantly. As an illustration, we calculate the monostatic RCS from the VFY218 at 100 MHz for VV polarization. Zero degree corresponds to incidence angle on the nose. In Figure 5, we plot the number of iterations for different incident angles using three kinds of initial guesses. The first case, which uses zero as the initial guess for all angles, needs about 85 iterations on the average for each angle. The second case, which uses the solution of the previous angle (2° step size) as the initial guess for the next angle, needs about 65 iterations per angle. The third case, which uses the phase-corrected solution of the previous angle as the initial guess for the next angle, needs only about 30 iterations per angle.

3.4. Approximation of Bistatic RCS to Monostatic RCS

For iterative solutions of monostatic RCS, different incident angles require different iterative solutions. Once the current distributions are found, calculating the RCS for one angle needs only $O(N)$ operations. Calculating the bistatic RCS is much less time-consuming than calculating the monostatic RCS. Thus, one advance feature in FISC is the calculation of the monostatic RCS using the bistatic RCS [18]. At 100 MHz, we need the monostatic RCS at 181 angles on the horizontal plane for the AZ angle from 0° to 180° for the VFY 218. We find the current distribution for 19 angles, and then use the approximation of bistatic RCS to monostatic RCS to calculate the monostatic RCS at all 181 points. In Figure 6, we plot the monostatic RCS of the VFY 218 at 100 MHz for HH polarization. The solid line is the monostatic RCS calculated using FISC without the approximation of bistatic RCS to monostatic RCS, the dashed line is the monostatic RCS calculated using FISC with the approximation, which needs only 1.5 hours on a DEC Alpha workstation.

4. Conclusions

FISC (Fast Illinois Solver Code) is designed to compute RCS of a target described by a triangular facet file. The MLFMA has been implemented to speed up the matrix-vector multiplies. Both the memory requirements and the CPU time per iteration are of $O(N\log N)$. Using a block diagonal preconditioner, near singularity extraction, phase corrected previous solution for the initial guess, and the approximation of bistatic RCS to monostatic RCS, we can solve for the electromagnetic scattering by large complex 3D objects like an aircraft (VFY 218) and automobiles on a small computer.

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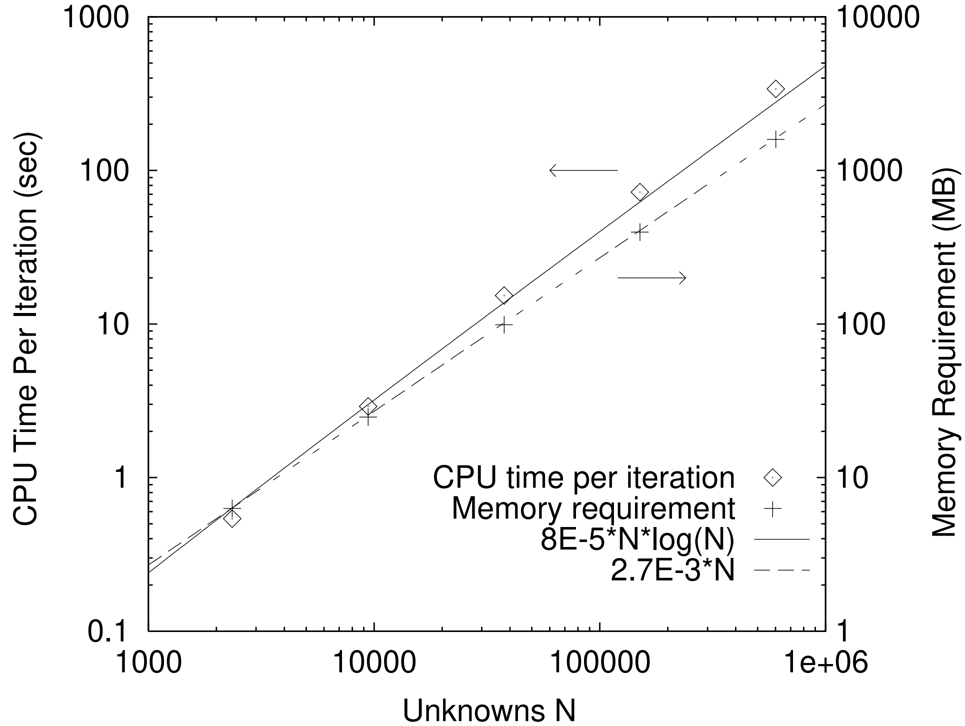

Figure 1. CPU time per iteration and memory requirements (points) as functions of number of unknowns for FISC. Two curves, $8 \times 10^{-5} N \log N$ and $2.7 \times 10^{-3} N$, are also plotted for comparison.

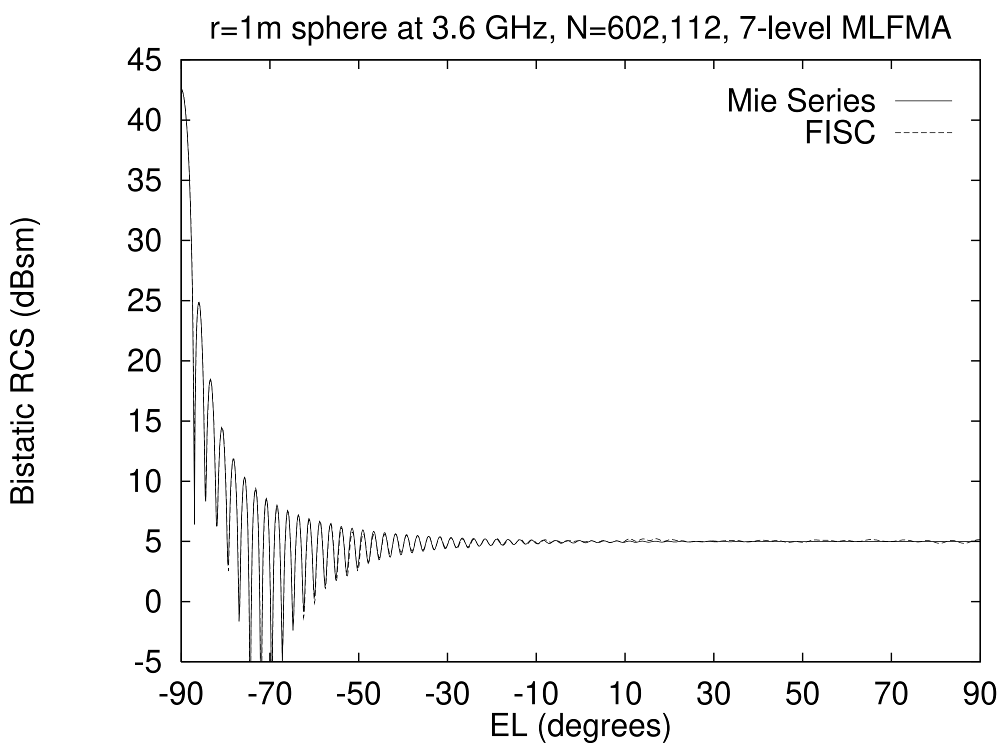

Figure 2. Validations of CFIE with MLFMA against the Mie series of the bistatic RCS of a metallic sphere of radius 1 m at 3.6 GHz for $\theta\theta$ polarization. A total of 602,112 unknowns with a seven-level MLFMA is used.

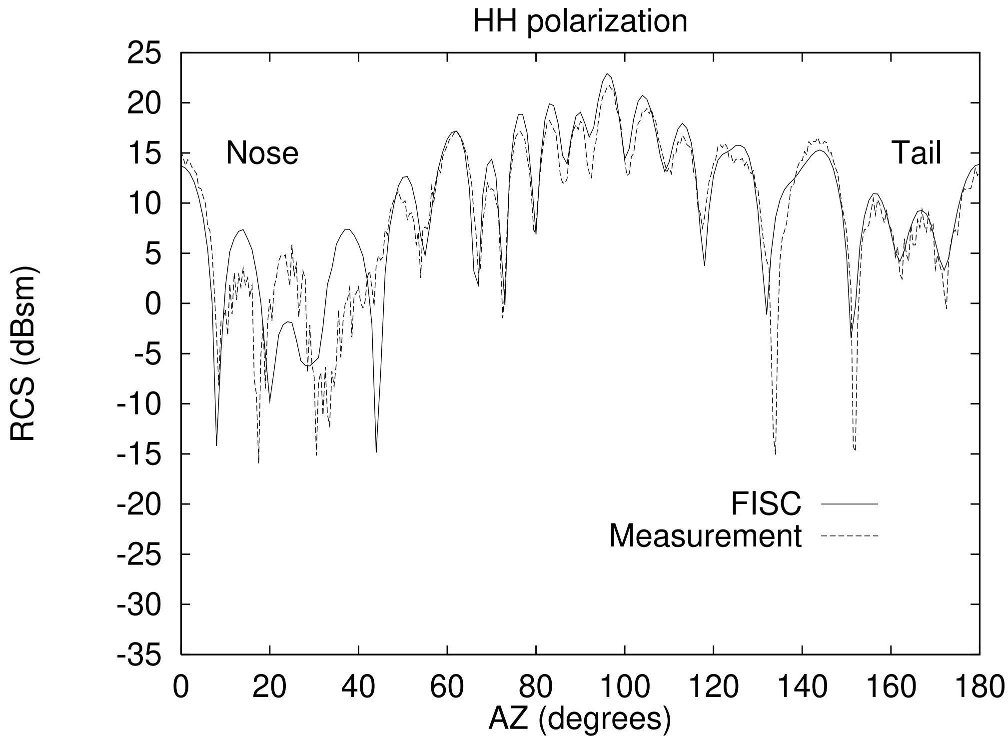

Figure 3. The monostatic RCS of the VFY 218 at 100 MHz. The measurement data are from Naval Air Warfare Center [20] (HH polarization).

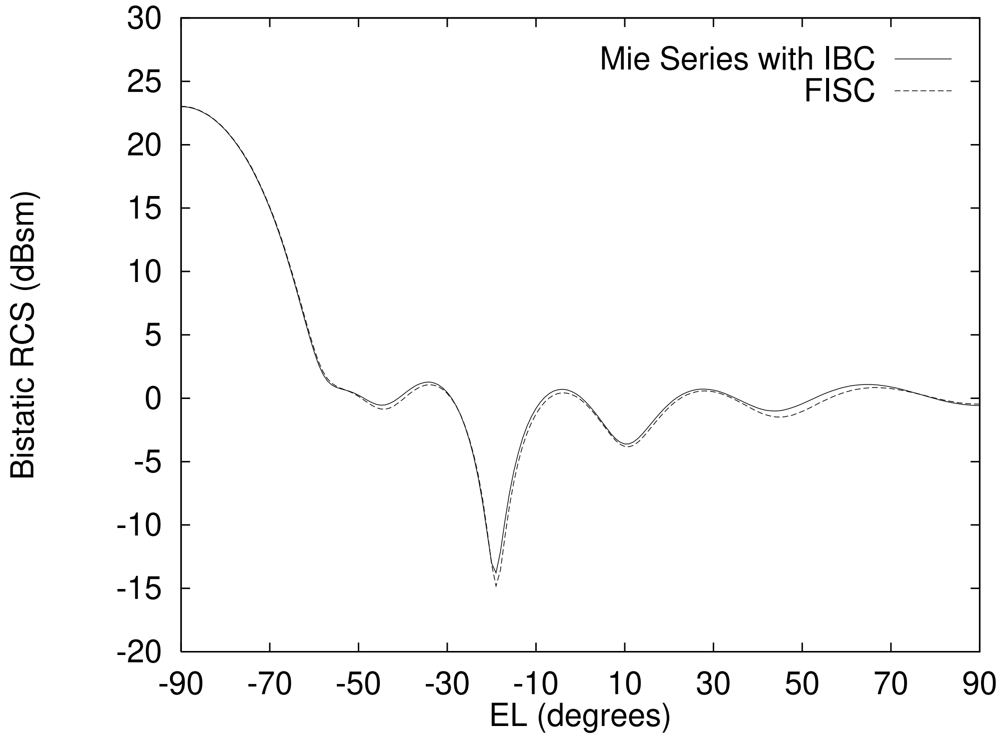

Figure 4. Bistatic RCS for a sphere with 1 m radius at 300 MHz as functions of elevation angle $(90 - \theta)$. The impedance of impedance boundary condition is $100 + j37.7$ ohms.

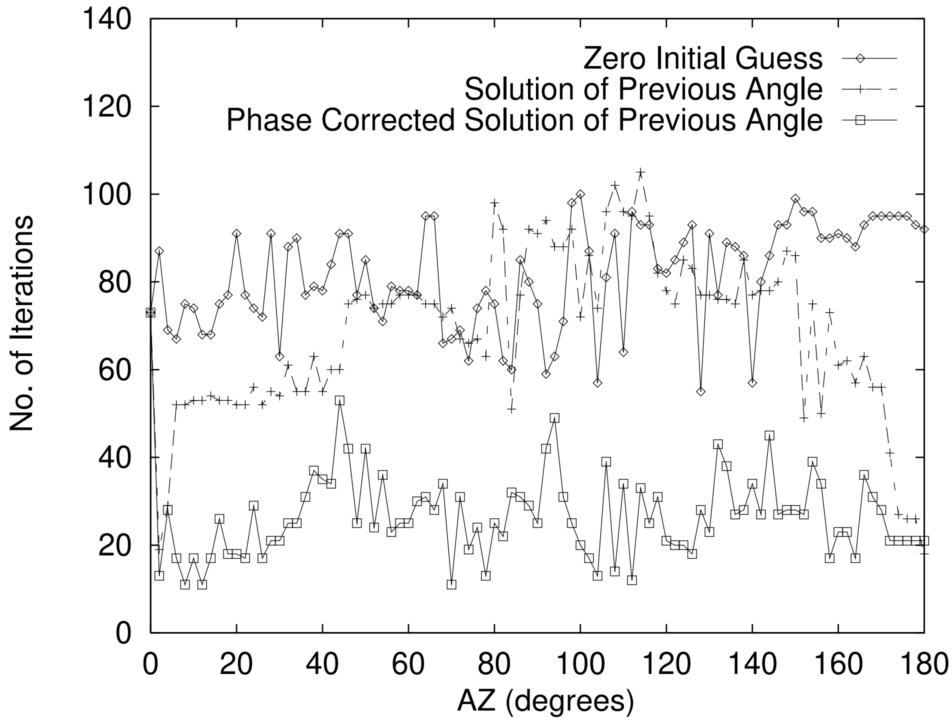

Figure 5. No. of iterations as functions of incident angles for different initial guesses: using zero initial guess for all angles, using the solution of the previous angle for the next angle with/without phase corrections.

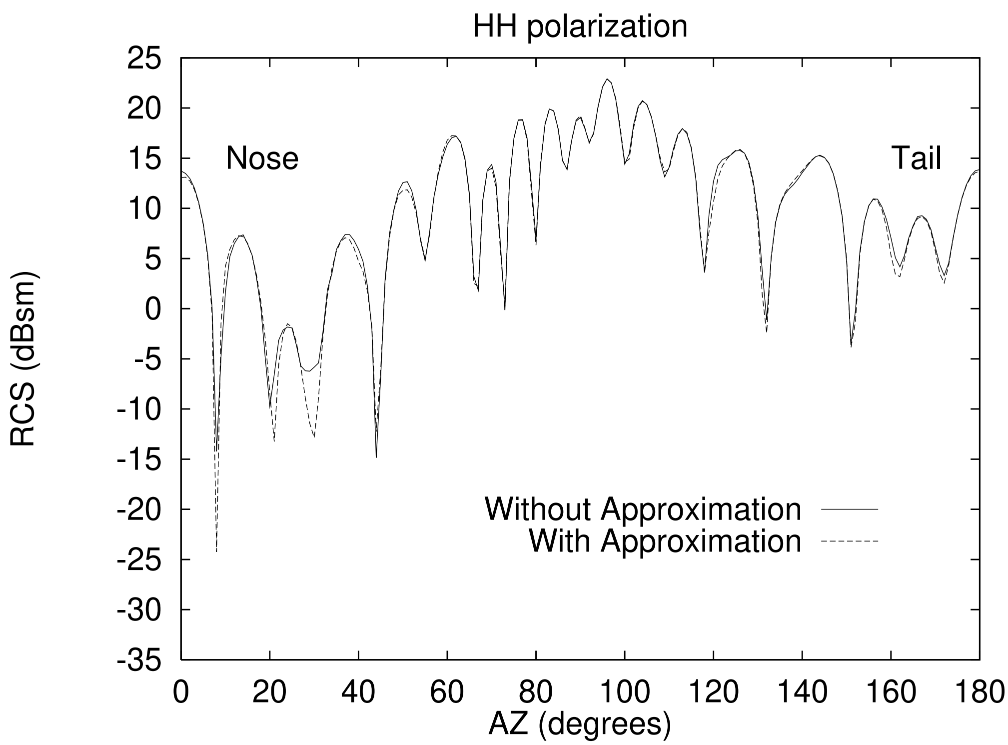

Figure 6. The monostatic RCS of the VFY 218 at 100 MHz. The numerical results with/without the approximation of bistatic RCS to monostatic RCS are calculated using FISC (HH polarization).